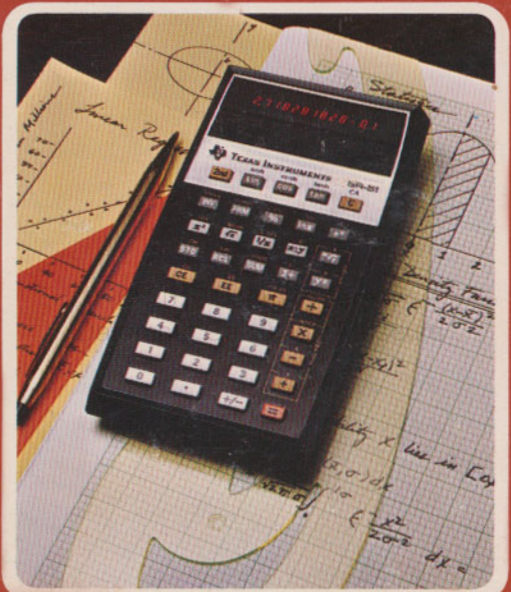
Texas Instruments
super slide-rule calculator
SR-51



OWNER'S MANUAL

KEY INDEX

This index permits quick page location of the description (dark no.) and one sample problem (light no.) for each key.

Toll-Free Telephone Assistance

For assistance with your SR-51 calculator, call one of the following toll-free numbers if necessary:

800-527-4980 (within all continental states except Texas)
800-492-4298 (within Texas)

See page 122 and back cover for further information on service.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

TABLE OF CONTENTS (continued)

TABLE OF CONTENTS (continued)

Section *Page*

SECTION I
DESCRIPTION

Your SR-51 is a powerful computational tool capable of solving a wide variety of problems. It has been designed for those who require an accurate, reliable, and versatile slide-rule calculator.

Because your SR-51 uses the algebraic mode of entry, you can *probably* already perform most calculations. However, to assist you in obtaining the most benefit from your SR-51, we have prepared this Owner's Manual and an Operating Guide to carry with you in the calculator carrying case. On the back of the calculator is a table listing the preprogrammed conversion codes and concise instructions on how to use them.

The SR-51 Operating Guide outlines briefly the operation of each key. It also gives the keystroke sequence when function keys are used together.

The fundamental operations your SR-51 can perform are explained in detail in the first three sections of this manual. In Section IV we have provided you with a wide variety of sample problems which are designed to suggest some of the many diverse applications possible with your SR-51.

The appendices detail supplementary information that will enhance your operation of the SR-51. Appendix A discusses register level processing. Simple linear correlation is discussed in Appendix B. Appendix C provides a review of inverse functions. The constants used internally by the conversion routines are found in Appendix D. A list of commonly used financial equations and some frequently used mathematical expressions are given in Appendices E and F, respectively.

The concluding material in this manual is devoted to providing information about operating your SR-51 either on battery pack or on house current, recharging the batteries,and obtaining service. Information on your SR-51 warranty and the warranty card are given on the back cover. Be sure to read this important information carefully.

FEATURES

Second Function – Your SR-51 uses dual function keys to expand the number of calculator functions without increasing the number of keys or calculator size.

Algebraic Entry – The SR-51 uses the algebraic entry method to simplify calculator operation. For simple problems, the numbers and algebraic functions are entered into the calculator in the same sequence as they are stated algebraically. For example, the problem of adding 15 to 25 and then subtracting 30 is normally stated as:

$$25 + 15 - 30 = 10$$

and is entered as:

25 [+] 15 [−] 30 [=] 10

Sum of Products – The SR-51 provides sum-of-product capability without use of special keys. For example, the expression:

$$(2 \times 3) + (4 \times 5) + (6 \times 7) = 68$$

is entered as:

2 [×] 3 [+] 4 [×] 5 [+] 6 [×] 7 [=] 68

Similar calculations, such as sum (or difference) of quotients, powers, roots, factorials, etc., are entered in the same straightforward manner.

Accuracy – Calculations are carried to 13 significant digits internally. In floating-point mode, answers are rounded off to 10 significant digits. For maximum accuracy, the SR-51 uses all 13 significant digits for subsequent calculations. The displayed number is accurate to within ±1 in the least significant digit.

Fixed Point—Calculated results may be displayed with 0 to 8 decimal places. Regardless of fixed-point location, your SR-51 continues to calculate all results internally to 13 significant places.

Scientific Notation—Your SR-51 computes and displays numbers as large as $\pm 9.999999999 \times 10^{99}$ and as small as $\pm 1 \times 10^{-99}$. Answers are automatically converted to scientific notation when the calculated answer is greater than 10^{10} or less than 10^{-10}. Moreover, your SR-51 allows you to remove a number from scientific notation during calculations.

Automatic Clearing—Your SR-51 calculator automatically clears itself. When the [=] key is pressed to complete the evaluation of an expression, the calculation is completed, the answer is displayed, and the calculator is cleared for the start of a new problem. It is not necessary to press the clear key between calculations except for some statistical functions and linear regression.

Calculation Time—Most calculations except large factorials are performed in a fraction of a second.

Memories—Three data registers for data calculation and three memory registers for data storage.

Fully Portable—Extremely lightweight. Battery or AC operated.

Long Life—Solid state components, integrated circuits, and light emitting diode display provide dependable operation and long life.

Battery Pack—The SR-51 comes complete with a *fast-charge* rechargeable battery pack, model BP-1. Under normal use, the battery pack will provide 3 to 6 hours of operation without recharging. About 4 hours of recharging will restore full charge. Spare and replacement battery packs can be purchased directly from a Texas Instruments Consumer Services Facility as listed on the back cover.

AC Adapter/Charger – Battery pack recharge or direct operation from standard voltage outlets is easily accomplished with the AC Adapter/Charger model AC9200 or AC9130 included with the SR-51 (also used with the SR-10, SR-11, SR-16, and SR-50). The SR-51 cannot be overcharged; it can be operated indefinitely with the adapter/charger connected.

DISPLAY DESCRIPTION

In addition to power-on indication and numerical information, the display provides indication of a negative number, decimal point, overflow, and error.

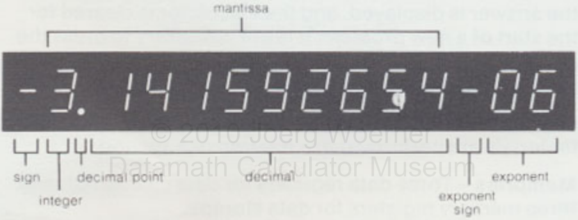


Figure 1

Minus Sign – Appears to the left of the 10-digit mantissa to indicate negative numbers, and appears to the left of the exponent (right of the mantissa) to indicate negative exponents (See Figure 1).

Decimal Point – Automatically assumed to be to the right of any number entered unless placed in another position with the [•] key. When entering numbers, the decimal will not appear until [•] is pressed.

Overflow and Error Indication – The display will flash for the following reasons:

1. Entry or calculation result outside the range of the calculator, $\pm 1 \times 10^{-99}$ to $\pm 9.999999999 \times 10^{99}$
2. Factorial of any number except a positive integer or zero
3. Inverse of a trigonometric or hyperbolic function with an invalid value for the argument, such as $\sin^{-1} x$ with x greater than 1
4. Square root or logarithm of a negative number
5. Raising a negative number to any power or taking the root of a negative number (The y^x and $\sqrt[x]{y}$ functions use the logarithmic routine which is undefined for negative numbers.)
6. Pressing [+], [−], [×], [÷], [y^x], [$\sqrt[x]{y}$] or [Δ%] during a linear regression routine
7. Entry of less than two data points for linear regression or variance and standard deviation calculations
8. Entry of more than 99 data points for linear regression.

Indication Removal – The flashing display caused by overflow or error will continue during subsequent calculation until the [C] key is pressed.

SECTION II
OPERATING INSTRUCTIONS AND EXAMPLES

ON/OFF SWITCH

Located on the top right front surface of the calculator. Sliding it to the right applies power, and sliding it to the left removes power from the calculator. The power-on condition is indicated by a number in the display. NOTE: After turning calculator on and before performing the first calculation, press [2nd] [CA] (see page 8).

SECOND FUNCTION KEY

The SR-51 keyboard, which is illustrated in Figure 2, shows that almost all keys perform two operations. The first function of a key is printed *on* the key, while the second function of a key is written on the overlay *above* the key. The [2nd] key in the upper left-hand corner of the keyboard places the calculator into second-function mode. To execute a second-function command, press [2nd], then press the key immediately below the desired second function.

For example, to find 5!, enter 5, press [2nd], then press [x:y]. The answer, 120, shows in the display. When [2nd] is pressed twice in succession, the calculator returns to first-function operation. This feature allows you to cancel unintentional second-function instructions. Also, the execution of any second-function command returns the calculator to first-function operation.

Symbolically, first-function operation will be denoted by black type on white background such as [x^2]. Second-function operation will be denoted by the second-function key symbol followed by a key symbol with white type on black background. For example, to show the factorial function, we denote this as [2nd] [x!].

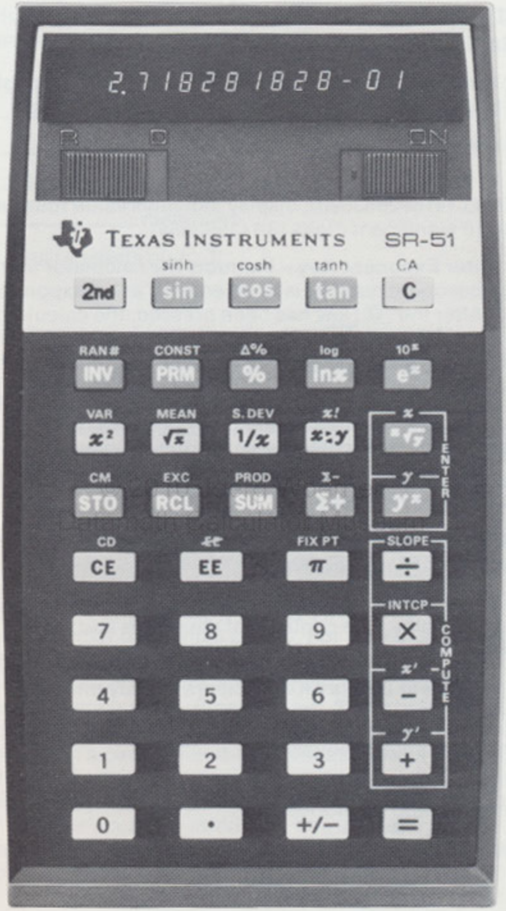

2.718281828-01
ON
TEXAS INSTRUMENTS
SR-51
sinh
cosh
tanh
CA
2nd
sin
cos
tan
C
RAN#
CONST
Δ%
log
INV
PRM
%
VAR
MEAN
S.DEV
x!
1/x
CM
EXC
PROD
STO
RCL
SUM
Σ+
ENTER
CD
FIX PT
SLOPE
CE
EE
π
÷
INTCP
×
COMPUTE
7
8
9
4
5
6
−
1
2
3
+
0
•
+/−
=

Figure 2

DATA ENTRY
Data Entry Keys

[0] **through** [9] **Digit Keys** – Enter numbers 0 through 9 to a limit of a 10-digit mantissa and a 2-digit exponent.

[•] **Decimal Point Key** – Enters a decimal point.

[π] **Pi Key** – Enters the value of pi (π) to 13 significant digits (3.141592653590); display indicates value rounded off to 10 significant digits (3.141592654).

[EE] **Enter Exponent Key** – Instructs the calculator that the subsequent number is to be entered as an exponent of 10. After the [EE] key has been pressed, the calculator will display all further results in scientific notation until [C], [2nd] [CA] or [2nd] [EE] [=] is pressed.

[+/–] **Change Sign Key** – Instructs the calculator to change the sign of the number appearing in the display. When pressed after [EE], changes sign of the exponent.

Data Removal Keys

[CE] **Clear Entry Key** – Clears last numeric entry when made with [0] - [9] keys.

[C] **Clear Key** – Clears current calculation in progress and the display. The contents of memories and the location of fixed decimal point are not affected.

[2nd] [CD] **Clear Display Key** – Clears display only.

[2nd] [CM] **Clear Memory Key** – Clears data in all three memories simultaneously.

[2nd] [CA] **Clear All Key** – Clears all calculator registers, operations and memories.

The following diagram will be useful in understanding the hierarchy of action of the various clear keys.

CE ______________________ number entry

CD ______________ any number in display plus ↑

C _____ calculations in progress plus ↑

CM __________ memories ↑

CA ___ all registers plus ↑

The difference between the CE and CD keys is that the CE key only clears displayed numbers entered from the keyboard while CD clears any number in the display register whether a number entry or a calculated result. Notice that if you enter a number and press a function key, e.g., the + key, CE will not clear the display. You must use CD. The constant pi (π) is removed from display by using CD.

Fixed Point Key – Calculated results in your SR-51 may be displayed with zero to eight decimal places. At any point during a calculation you may select the location of the decimal point by pressing 2nd Fix Pt, then entering the desired number of decimal places. For zero digits to the right of the decimal point, enter 0; to obtain one digit enter 1, and so forth. The largest fixed-point location that you may select is eight. Pressing 2nd Fix Pt 9 or 2nd CA restores the calculator to floating point. Remember that even though a certain fixed-point location for the display has been selected, you may enter numbers with up to ten decimal places and your SR-51 will continue to calculate with 13 place accuracy internally. The display, however, will only show the number of decimal places selected. As in floating point mode, the least significant digit in the display is rounded. For example, find the area of a circle with radius 2.314679 to three decimal places.

Enter	Press	Display	Comments
	2nd Fix Pt. 3	0.000	Calculator in Fix Pt 3
2.314679	x^2 ×	5.358	Radius squared
	π =	16.832	Area to 3 decimal places

Now press:

	2nd Fix Pt. 9	16.83183308	Area calculated internally to 13 decimal places. Display rounded to 10.

Negative Number Entry – A negative sign is assigned to a number by pressing the +/– key directly after entering the number.

For example, to enter –125:

Enter	Press	Display
125	+/–	–125

Scientific Notation – Any number can be entered into the SR-51 in scientific notation – that is, as a number (mantissa) multiplied by 10 raised to some power (exponent). For example, 1000 can be written as $1. \times 10^3$.

Enter	Press	Display
1	EE	1 00
3		1 03

NOTE: The last two digits on the right side of the display are used to indicate exponents.

Very large and very small numbers must be entered in scientific notation. For example, 120,000,000,000 is written as 1.2×10^{11}.

Enter	Press	Display
1.2	[EE]	1.2 00
11		1.2 11

In both of these examples, the exponent indicates how many places the decimal should be moved to the right. If the exponent is negative, the decimal should be moved to the left. For example, $1.2 \times 10^{-11} = 0.000000000012$.

Enter	Press	Display
1.2	[EE]	1.2 00
11	[+/−]	1.2 −11

To change the mantissa or its sign after the [EE] key has been pressed, simply press the [·] key and make the appropriate entry on the keyboard. To change the exponent or its sign, simply press the [EE] key again and make the appropriate entry.

Enter	Press	Display	Comments
1.327	[EE]	1.327 00	
21		1.327 21	
	[·]	1.327 21	To change mantissa
65	[+/−]	−1.32765 21	
	[EE]	−1.32765 21	To change exponent
22	[+/−]	−1.32765 −22	

Data in scientific notation form may be entered intermixed with data in normal form. The calculator will convert the entered data for proper calculation.

For example: $12575 + 3.2 \times 10^3 + 2855 = 1.863 \times 10^4$.

Enter	Press	Display
	[C]	0
12575	[+]	12575.
3.2	[EE]	3.2 00
3	[+]	1.5775 04
2855	[=]	1.863 04

Note that pressing [EE] also instructs the calculator to use only the number in display for subsequent calculations. In effect this truncates any internally held digits not shown in the display.

For example, press [π] [EE] − [π] [=]. The answer displayed is 4.1×10^{-10}.

The reason you do not get zero ($\pi - \pi$) is because [EE] effectively truncated the three internally stored digits of the first π entered. In this example only *three* internally held digits were discarded. If the calculator had been placed in fixed-point mode when [EE] was pressed, all internal digits not appearing in the display would have been discarded for subsequent calculations.

Scientific Notation Removal—Numbers appearing in the display in scientific notation can be converted back to standard form with appropriate decimal point positioning by using the [~~EE~~] key. For example, to convert 1×10^3 to 1000:

Enter	Press	Display
1	[EE]	1 00
3		1 03
	[2nd] [~~EE~~] [=]*	1000.

The [~~EE~~] key removes from scientific notation only those numbers whose exponent has an absolute value of less than 10. Exponents whose absolute value is 10 or greater remain in scientific notation.

*For displayed numbers which are calculated results, the [=] key need not be pressed.

Error Correction—Incorrect number key entries are corrected by pressing the CE key before pressing the next function key in the calculation.

Example: $3 \times \cancel{4}\ 5 = 15$

Enter	Press	Display	Comments
3	×	3.	
4		4	Error
	CE	0	Correction
5	=	15.	

Pi is entered as a calculated value and is not cleared by this key. If the π key is inadvertently pressed, it can be nullified by entering the correct number. For example, in entering 2.395, the π key might be pressed accidentally instead of the 9 key.

Enter	Press	Display	Comments
2.3		2.3	
	π	3.141592654	Error
2.395		2.395	Correction

Arithmetic Function Keys

+ **Add Key**—Instructs the calculator to add to the previous number or result the next entered number or result.

− **Subtract Key**—Instructs the calculator to subtract from the previous number or result the next entered number or result.

× **Multiply Key**—Instructs the calculator to multiply the displayed number by the next entered quantity.

÷ **Divide Key**—Instructs the calculator to divide the displayed number by the next entered quantity.

[=] **Equals Key**—Instructs the calculator to complete the calculation of all the previously entered algebraic functions. As the lowest level operator in the calculator hierarchy, this key may be used to complete both intermediate results and final results.

Addition and Subtraction

Example: $12.32 - 7 + 1.6 = 6.92$

Enter	Press	Display
12.32	[−]	12.32
7	[+]	5.32
1.6	[=]	6.92

Multiplication and Division

Example: $(4 \times 7.3) \div 2 = 14.6$

Enter	Press	Display
4	[×]	4.
7.3	[÷]	29.2
2	[=]	14.6

Error Correction

The SR-51 has been designed to facilitate correction of the most common function key errors. If a [+] key is inadvertently pressed instead of a [−] key (or vice versa), the error is corrected by simply pressing the correct function key.

Example: $5 \cancel{+} - 2 = 3$

Enter	Press	Display	Comments
5	[+]	5.	Error
	[−]	5.	Correction
2	[=]	3.	

The calculator automatically inserted a zero to complete the erroneous function key entry. Thus, the above key sequence is calculated as $5 + 0 - 2 = 3$. The sequence 5 $\boxed{-}$ $\boxed{+}$ 2 $\boxed{=}$ is calculated as $5 - 0 + 2 = 7$. If a $\boxed{\times}$ or $\boxed{\div}$ key is pressed instead of any other arithmetic function key, the error is corrected simply by pressing the correct key.

Example: $5 \cancel{\div} + 2 = 7$

Enter	Press	Display	Comments
5	$\boxed{\div}$	5.	Error
	$\boxed{+}$	5.	Correction
2	$\boxed{=}$	7.	

In this case, the calculator completed the erroneous function by inserting a one before entering the $\boxed{+}$ function. Thus, this key sequence is calculated as $(5 \div 1) + 2 = 7$. The key sequence 5 $\boxed{\times}$ $\boxed{+}$ 2 $\boxed{=}$ is calculated as $(5 \times 1) + 2 = 7$.

NOTE: An accidental double entry of any arithmetic function key is automatically corrected because the SR-51 inserts a zero between two $\boxed{+}$ or $\boxed{-}$ operations and a one between two $\boxed{\times}$ or $\boxed{\div}$ operations.

If a $\boxed{+}$ or $\boxed{-}$ key is inadvertently pressed instead of a $\boxed{\times}$ or $\boxed{\div}$ key, you can correct the error by pressing the $\boxed{=}$ key and then the correct function key.

Example: $5 \cancel{+} \times 2 = 10$

Enter	Press	Display	Comments
5	$\boxed{+}$	5.	Error
	$\boxed{=}$ $\boxed{\times}$	5.	Correction
2	$\boxed{=}$	10.	

Again the calculator automatically inserted a zero after the [+] key but the [=] had to be pressed to complete the erroneous [+] function because of the sum-of-products capability. Thus, the above sequence is calculated as $(5 + 0) \times 2 = 10$. If the [=] key had been omitted, the calculator would have calculated the data as $5 + (0 \times 2) = 5$.

Pressing the [=] key immediately after any arithmetic function key effectively replaces the arithmetic operation by an [=] function. Thus, 5 [+] 2 [+] [=] is calculated as $5 + 2 + 0 = 7$ and 5 [×] 2 [×] [=] is calculated as $5 \times 2 \times 1 = 10$. Thus, the [=] key can be used to complete any arithmetic function key that was erroneously pressed.

SINGLE-VARIABLE FUNCTION KEYS

The single-variable function keys operate only on the display register which may contain either a number entry or a calculated result. They do not complete any previously entered function.

Special Functions

[x^2] **Square Key**-Instructs the calculator to find the square of the number displayed.

[$\sqrt{x}$] **Square Root Key**-Instructs the calculator to find the square root of the number displayed.

[1/x] **Reciprocal Key**-Instructs the calculator to find the reciprocal of the number displayed.

[2nd] [x!] **Factorial Key**-Instructs the calculator to find the factorial of the number displayed. The largest factorial the SR-51 can compute without an overflow condition is 69! .

Squares

Example: $(4.2)^2 = 17.64$

Enter	Press	Display
4.2	$\boxed{x^2}$	17.64

Square Roots

Example: $\sqrt{6.25} = 2.5$

Enter	Press	Display
6.25	$\boxed{\sqrt{x}}$	2.5

Example: $\sqrt{4} + \sqrt{9} = 5$

Enter	Press	Display
4	$\boxed{\sqrt{x}}$ $\boxed{+}$	2.
9	$\boxed{\sqrt{x}}$	3.
	$\boxed{=}$	5.

Reciprocals

Example: $\frac{1}{3.2} = 0.3125$

Enter	Press	Display
3.2	$\boxed{1/x}$	0.3125

Factorials

Example: $7! = 5040$

Enter	Press	Display
7	$\boxed{2nd}$ $\boxed{x!}$	5040.

Example: $\frac{60}{4!} = 2.5$

Enter	Press	Display
60	÷	60.
4	2nd x!	24.
	=	2.5

When the factorial of a noninteger number is computed, only the whole number is considered and the display flashes indicating the fractional part was ignored. The C key must be pressed to remove the flashing condition of the display.

Example: 7.3! = 5040

Enter	Press	Display	Comments
7.3	2nd x!	5040.	Flashing display

The factorial function operates only on the displayed number and numbers not displayed are ignored.

Trigonometric and Hyperbolic Functions

Deg/Rad Switch – Located on the top left front surface of the calculator. The calculator interprets a displayed angle as being in degrees if the switch is to the right (D) and in radians if it is to the left (R).

sin **Sine Key** – Instructs the calculator to determine the sine of the displayed angle.

cos **Cosine Key** – Instructs the calculator to determine the cosine of the displayed angle.

tan **Tangent Key** – Instructs the calculator to determine the tangent of the displayed angle.

2nd sinh **Hyperbolic Sine Key** – Calculates the hyperbolic sine of the number displayed.

2nd cosh **Hyperbolic Cosine Key** – Calculates the hyperbolic cosine of the number displayed.

[2nd] [tanh] **Hyperbolic Tangent Key** – Calculates the hyperbolic tangent of the number displayed.

[INV] **Inverse Key** – Used prior to trigonometric and hyperbolic functions to calculate inverse functions. Also used with list of 20 conversions to reverse order of conversion. Cancels inverse instruction when pressed twice in succession. For example, the proper key sequence for $\sin^{-1}$ is [INV] [sin], while the correct key sequence for $\tanh^{-1}$ is [INV] [2nd] [tanh].

Trigonometric Calculations

The SR-51 will calculate trigonometric values for angles greater than 360 degrees (2π radians) or less than −360 degrees (-2π radians). As long as the trigonometric function is displayed in normal form rather than in scientific notation, all 10 displayed digits are accurate for the range −36000 to 36000 degrees (-200π to 200π radians). In general, the accuracy decreases one digit for each decade outside this range. If the magnitude of the angle is 1.001×10^{14} degrees or larger, the SR-51 interprets it as 0 degrees.

Throughout the manual, the notation Angle: Deg means set the Deg/Rad switch to D and Angle: Rad means set the Deg/Rad switch to R.

Example: sin 30° = 0.5

Angle:Deg

Enter	Press	Display
30	[sin]	0.5

Example: 14.3 tan 1.385 = 76.0783255

Angle:Rad

Enter	Press	Display
14.3	[×]	14.3
1.385	[tan] [=]	76.0783255

Example: $\sin^{-1} 0.5 = 30°$

Angle:Deg

Enter	Press	Display
0.5	INV sin	30.

Example: $\frac{\pi}{4} + \tan^{-1} 1 = 1.570796327$

Angle:Rad

Enter	Press	Display
π	÷	3.141592654
4	+	.7853981634
1	INV tan =	1.570796327

Hyperbolic Calculations

Example: $\tanh 6.43 = 0.9999948$

Enter	Press	Display
6.43	2nd tanh	0.9999948

Example: $\sinh^{-1} 0.886 = 0.7984245338$

Enter	Press	Display
.886	INV 2nd sinh	.7984245338

Logarithmic Functions

The logarithmic function keys provide for processing of logarithmic quantities to base e or base 10.

lnx **Natural Logarithm Key** – Calculates the natural logarithm of the number displayed. $x \geq 0$.

[e^x] **e to the x Power Key** – Raises e to the power shown in display.

[2nd] [log] **Common Logarithm Key** – Calculates the common logarithm of the number displayed. $x \geq 0$.

[2nd] [10^x] **Common Antilogarithm Key** – Raises 10 to the power shown in display.

Example: ln 5.4 = 1.686398954

Enter	Press	Display
5.4	[lnx]	1.686398954

Example: 31.78 + 4 ln 19.3 = 43.62042038

Enter	Press	Display
31.78	[+]	31.78
4	[×]	4.
19.3	[lnx]	2.960105096
	[=]	43.62042038

Example: $e^{3.8}$ = 44.70118449

Enter	Press	Display
3.8	[e^x]	44.70118449

Example: log 1573 = 3.196728723

Enter	Press	Display
1573	[2nd] [log]	3.196728723

Example: $10^{3.2}$ = 1584.893192

Enter	Press	Display
3.2	[2nd] [10^x]	1584.893192

TWO-VARIABLE FUNCTION KEYS

The two-variable function keys process two numbers in a single operation. The numbers can be a keyboard entry, a calculated result, a stored quantity, or a combination of the two.

Y [y^x] X [=] y to the x Power Key Sequence – Raises y to the power x. $y \geq 0$

Y [$\sqrt[x]{y}$] X [=] the x^{th} Root of y Key Sequence – Finds the x^{th} root of y. $y \geq 0$

[x:y] X Exchange Y Key – Exchanges the contents of the X and Y registers. It exchanges factors in a multiplication and it exchanges divisor and dividend in a division. In a [y^x] or [$\sqrt[x]{y}$] routine it exchanges base and exponent or base and root respectively. It is also used to make data entries for operations requiring more than a single data point, e.g., polar-rectangular, ratio to dB conversions, and permutation.

n [x:y] r [PRM] Permutation Key Sequence – Determines the number of permutations of n items taken r at a time; $0 \leq n \leq 69$, $r < n$, n and r integers. The formula for the number of permutations of n things taken r at a time is given by $P\binom{n}{r} = \frac{n!}{(n-r)!}$. When $r = n$, the number of permutations is easily calculated as n!. We may use permutations to find the number of combinations of n things taken r at a time which is defined as $P\binom{n}{r} \div r!$.

X_1 [2nd] [Δ%] X_2 [=] Delta Percent Key Squence – Calculates the percentage change between X_1 and X_2

defined as

$$\frac{X_2 - X_1}{X_1} \times 100$$

Powers

Example: $(8)^3 = 512$

Enter	Press	Display
8	y^x	8.
3	=	512.

Example: $(2)^{3+4} = (2)^7 = 128$

Enter	Press	Display	Comments
3	+	3.	
4	= y^x	7.	
2		2	
	x:y	7.	Exchange x and y
	=	128.	

Example: $34.7 + (8.7)^{2.6} = 311.8724475$

Enter	Press	Display
34.7	+	34.7
8.7	y^x	8.7
2.6	=	311.8724475

The following complex functions can be calculated easily with the SR-51; $y^{1/x}$, $y^{\sqrt{x}}$, $y^{\sin x}$, $y^{\ln x}$, etc.

Example: $4.2^{\ln 3.7} = 6.537587302$

Enter	Press	Display
4.2	y^x	4.2
3.7	lnx	1.30833282
	=	6.537587302

Roots

Example: $\sqrt[1.3]{4.8} = 3.342194507$

Enter	Press	Display
4.8	[ˣ√y]	4.8
1.3	[=]	3.342194507

Example: $\sqrt[4.7]{215} + 5.86 = 8.995187378$

Enter	Press	Display
215	[ˣ√y]	215.
4.7	[+]	3.135187378
5.86	[=]	8.995187378

Permutations

Calculate the number of permutations of 10 items taken 6 at a time.

Enter	Press	Display
10	[x:y]	
6	[PRM]	151200.

To now calculate the number of combinations of 10 items taken 6 at a time

Enter	Press	Display
	[÷]	151200.
6	[2nd] [x!]	720.
	[=]	210.

Delta Percent

What is the percent change from 5 to 3?

Enter	Press	Display	Comments
5	[2nd] [Δ%]	5.	
3	[=]	−40.	A decrease of 40%

PERCENT KEY

[%] **Percent Key** – Converts displayed number from a percentage to a decimal. When [%] is pressed after the arithmetic operations, add on, discount, and percentage may be computed as follows:

[+] n [%] [=] adds n% to the number displayed. For example, how much is paid for a $10 item when the sales tax is 5%?

Enter	Press	Display
10	[+]	10.
5	[%] [=]	10.5

[−] n [%] [=] subtracts n% from the number displayed. How much is paid for a $5 item discounted at 2%?

Enter	Press	Display
5	[−]	5.
2	[%] [=]	4.9

[×] n [%] [=] multiplies number in display times n%. What is 2.5% of 15?

Enter	Press	Display
15	[×]	15.
2.5	[%] [=]	0.375

[÷] n [%] [=] divides number in display by n%. 25 is 15% of what number?

Enter	Press	Display
25	[÷]	25.
15	[%] [=]	166.6666667

CONSTANT FUNCTION KEY

Repetitive calculations can be handled easily using the constant feature of the calculator. Entry of a constant arithmetic operation is simple and direct and includes the [+], [−], [×], [÷], [y^x], [$\sqrt[x]{y}$] and [Δ%] functions. To use the constant-mode feature,first enter the repetitive operation, then the constant, n, followed by [2nd] [CONST]. Repetitive calculations are completed by entering the variable and pressing [=].

[+] n [2nd] [CONST] adds n to each subsequent entry.

[−] n [2nd] [CONST] subtracts n from each subsequent entry.

[×] n [2nd] [CONST] multiplies each subsequent entry by n.

[÷] n [2nd] [CONST] divides each subsequent entry by n.

[y^x] n [2nd] [CONST] raises each subsequent entry to the power n, i.e., y^n.

[$\sqrt[x]{y}$] n [2nd] [CONST] takes the nth root of each subsequent entry, i.e., $\sqrt[n]{y}$.

[2nd] [Δ%] n [2nd] [CONST] calculates percentage change between n and each subsequent entry which is defined as

$$\frac{x-n}{n} \times 100.$$

Pressing [C] or entering any of the above functions removes the calculator from constant mode operation.

Of considerable value is the fact that you may interchange the entry order of constant and function, and arrive at the same result. This added feature of your SR-51 allows you to do complex combinations of arithmetic operations in constant mode. Combinations are of the form:

Enter	Calculate
A [+] B [×] [2nd] [CONST]	$(B \times S) + A$ where S is each subsequent entry with A and B constant
A [−] B [×] [2nd] [CONST]	$(B \times S) - A$
A [+] B [÷] [2nd] [CONST]	$\frac{S}{B} + A$
A [+] B [y^x] [2nd] [CONST]	$S^B + A$
A [+] B [$\sqrt[x]{y}$] [2nd] [CONST]	$\sqrt[B]{S} + A$
A [+] B [2nd] [Δ%] [2nd] [CONST]	$\frac{S - B}{B} \times 100 + A$

For example, to calculate $\sqrt[3]{S}$ for $S = 64$ and $S = 27$, first press [C], then:

Enter	Press	Display
	[$\sqrt[x]{y}$]	0.
3	[2nd] [CONST]	3.
64	[=]	4.
27	[=]	3.

RANDOM NUMBER KEY

[2nd] [RAN#] **Random Number Key** – Using this sequence of key strokes, your SR-51 generates a two-digit random number from 00 to 99. Each execution of this key sequence will produce a new two-digit random number. The display is leading zero suppressed; therefore, random numbers in the first decade (00-09) will contain only one digit (0-9).

MEMORY KEYS

Your SR-51 has three user-accessible memories. Because use of the memory keys does not interfere with calculations in progress, they may be used at any point in a calculation. The memory keys allow data to be stored and retrieved for additional flexibility in calculation. All memory related commands *must* be followed by the memory address n (1, 2, or 3).

[STO] **n Store Key** – Stores display into memory location n. Any previously stored data is cleared. This key does not affect the displayed number.

[SUM] **n Sum to Memory Key** – Algebraically sums display to the contents of memory n and stores result in memory n. This key does not affect the displayed number.

[2nd] [PROD] **n Product to Memory Key** – Multiplies contents of memory n by number displayed and stores result in memory n. This key does not affect the displayed number.

[RCL] **n Recall Key** – Displays data stored in memory location n without clearing the memory. Recalled number may be used as an entered quantity in any mathematical expression.

[2nd] [EXC] **n Exchange Key** – Exchanges contents of memory n with the display.

[2nd] [CM] **Clear memory Key** – Clears data of all three memories simultaneously.

Access to the three memories is restricted during mean, variance, standard deviation and linear regression routines. You cannot use them during these calculations.

Although further examples will be given later, here are some of the ways the memories can be used.

Storing Data

Example: Store 7. in Memory 1
Store 8. in Memory 2
Store 9. in Memory 3

Enter	Press	Display
7	STO 1	7.
8	STO 2	8.
9	STO 3	9.

Recalling Data

Example: Recall contents of all three memories after data of previous example has been entered.

Press	Display	Comments
RCL 1	7.	Memory 1 contents
RCL 2	8.	Memory 2 contents
RCL 3	9.	Memory 3 contents

Adding to Memory

Example: $(10+2)\times 3=36$
$+(10+3)\times 4=52$
$+(10+4)\times \underline{5=70}$
Total = 158

Enter	Press	Display	Comments
10	+	10.	
2	= ×	12.	
3	=	36.	
	STO 1	36.	Store 36 in Memory 1
10	+	10.	
3	= ×	13.	
4	=	52.	
	SUM 1	52.	Sum 52 to Memory 1
10	+	10.	
4	= ×	14.	
5	=	70.	
	SUM 1	70.	Sum 70 to Memory 1
	RCL 1	158.	Recall Memory 1

MEAN, VARIANCE, AND STANDARD DEVIATION KEYS

You can easily calculate the mean, variance and standard deviation of collected data with the special keys dedicated for this use on your SR-51. For your convenience we have provided you with the option of selecting N weighting or N-1 weighting in calculating the standard deviation. The former is generally used for describing populations, while the latter is customarily used for sample data and can be easily extended to calculate the standard error of the mean.

The Variance [VAR] is calculated using N weighting. Therefore, to find the standard deviation (using N weighting) you should take the square root of the variance. In an analogous manner, to find the variance (using N-1 weighting) you should square the Standard Deviation [S.DEV], which is calculated using N-1 weighting.

To Find / Method	Variance	Standard Deviation
N	[2nd] [VAR]	[2nd] [VAR] [$\sqrt{x}$]
N-1	[2nd] [S.DEV] [x^2]	[2nd] [S.DEV]

Entering Data

Press [2nd] [CM] or [2nd] [CA] before proceeding. To calculate the mean, standard deviation and variance of data $x_1, x_2, x_3, \ldots x_n$, enter x_1 and press [Σ+]. The number 1 will appear in the display to signify that a single data point has been entered. Continue for $x_2, x_3 \ldots x_n$. The numbers 2, 3, ... n will appear in the display after each successive entry to indicate the number of data points thus far entered. At any point in the data entering process, you may call for the mean, variance, or standard deviation. You may then resume data entry without destroying the previously entered data.

The key sequence x_i [Σ+] adds x_i to memory 1, adds x_i^2 to memory 2, and advances the count in memory 3 by 1. The key sequence x_i [2nd] [Σ-] decrements each memory using the same rule. Notice that this routine uses all three memories and data previously stored there cannot be recovered. You may, however, recall the contents of any memory and perform other calculator operations.

You may also compute the mean, variance and standard deviation by entering the x_i sum to memory 1, the x_i^2 sum to memory 2 and the number of data points to memory 3 and then using the desired function key. This feature is extremely useful for working with grouped data (see page 64).

[Σ+] **Sum Plus Key** – Enters displayed number into calculator as data point for calculation of mean, variance and standard deviation.

[2nd] [Σ-] **Sum Minus Key** – Removes displayed number as data point when calculating mean, variance and standard deviation. This key is used to correct erroneous entries.

[2nd] [MEAN] **Mean Key** – Calculates mean defined as:

$$\text{MEAN} = \bar{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

[2nd] [S.DEV] **Standard Deviation Key** – Calculates standard deviation of sample data using N-1 weighting:

$$\text{S. Dev.} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{X})^2}{N-1}}$$

[2nd] [VAR] **Variance Key** – Calculates population variance using N weighting:

$$\text{Variance} = \frac{\sum_{i=1}^{N} (x_i - \bar{X})^2}{N}$$

For example, calculate the mean, variance, and standard deviation of the test scores of the following six students assuming that the six students are the entire population. Test scores 70, 84, 95, 90, 88, 93.

Solution:

Enter	Press	Display	Comments
	2nd CM		Clear memories
70	Σ+	1.	
87	Σ+	2.	Entry error
87	2nd Σ-	1.	Entry correction
84	Σ+	2.	
⋮	⋮	⋮	⋮
93	Σ+	6.	Complete data entry
	2nd MEAN	86.66666667	Mean value
	2nd VAR	67.8888889	Variance
	$\sqrt{x}$	8.239471397	Standard deviation

LINEAR REGRESSION KEYS

Your SR-51 performs a least-squares linear regression on two-dimensional random variables (x_i, y_i) from a minimum of 2 to a maximum of 99 data points. Always press 2nd CA at the start of a problem. Normally enter x_i value first followed by y_i value. For trend analysis, enter only the y_i values in sequence $y_1, y_2, \ldots y_n$. Your SR-51 automatically assigns x_i the value i. Press 2nd CA to clear the linear regression routine. The form of the calculated linear regression curve is $f(x) = y = mx + b$ where m is the slope of the line and b is the y-intercept.

NOTE: Changing the decimal position (2nd Fix Pt n) after entering data points will produce erroneous results. Linear regression problems may be entered and calculated with fixed-point decimal by always starting the problem with 2nd CA 2nd Fix Pt n 2nd CM . The decimal position may not be changed until the final answer is obtained.

2nd x **Enter x Key** – Enters the number displayed as the x coordinate of an (x,y) data point.

2nd y **Enter y Key** – Enters the number displayed as the y coordinate of an (x,y) data point and forms a closed loop on data entry. The number of data points entered thus far will appear in the display.

2nd SLOPE **Slope Key** – Displays the slope, m, of the calculated linear regression curve.

2nd INTCP **Intercept Key** – Displays the y intercept, b, of the calculated linear regression curve.

2nd y' **Compute y Key** – Calculates f(x) where x is the value in display and f is the regression curve.

2nd x' **Compute x Key** – Calculates $f^{-1}(y)$ where y is the value in display and f is the regression curve.

The linear regression routine uses all nine calculator registers for processing data. It is, therefore, not possible to perform any operations which utilize these registers. Appendix A shows that only functions which operate on the X register may be used. The user may only use the following functions:

1. Trigonometric
2. Hyperbolic
3. Logarithmic
4. Factorial
5. Percent
6. x^2, $\sqrt{x}$, $1/x$

Of special interest is that by performing any of these functions on one or both elements of the random-variable pair, other types of correlation are possible.
For example, by taking the logarithm of one of the random variables before entering it as a data point, you may obtain a semi-logarithmic curve fit. Similar variations may be achieved by using the other functions.

As an example of a linear regression entry sequence assume that a company registers sales of 4, 7, 8, 7.5 and 10 million dollars during the past five years. What are the projected sales for the following year. We perform a trend analysis as follows:

Enter	Press	Display	Comments
	[2nd] [CA]		
4	[2nd] [y]	1.	Calculator automatically assigns x = 1
7	[2nd] [y]	2.	.
8	[2nd] [y]	3.	.
7.5	[2nd] [y]	4.	.
10	[2nd] [y]	5.	Calculator automatically assigns x = 5
6	[2nd] [y']	11.05	Projected sales for 6th year

Assume you are told that x varies logarithmically with y and that the following data exists:

x	y
0	1.
2	7.389056099
5	148.4131591
8	2980.957987
10	22026.46579

The linear regression form for a semi-logarithmic curve is:
$\ln y = \ln y_0 + mx$

Where

m = slope

$\ln y_0$ = y intercept

Enter	Press	Display	Comments
	2nd CA	0	
	2nd x	0.	Enter x_1
1	ln$x$ 2nd y	1.	Enter ln y_1
.	.	.	.
.	.	.	.
.	.	.	.
10	2nd x	10.	Enter x_5
22026.46579	ln$x$ 2nd y	10.	Enter y_5
	2nd SLOPE	1.	Value of m
	2nd INTCP e^x	1.	value of y_0

We find that:

$$\ln y = \ln 1 + x$$

or $$y = e^x$$

In order to find y, we have made use of the inverse function relationship of the natural logarithm and the exponential function. For a discussion of inverse functions, see Appendix C.

CONVERSION KEYS

Basic Conversions (Codes 00-16) Refer to the conversion codes shown in Table I.

n 2nd **Two-digit Code** – Converts n number of units in the left column to units in the center column of Table I.

n INV 2nd **Two-digit Code** – Converts n number of units in the center column to units in the left column of Table I.

TABLE I

From	To	Code
mils	microns	00
inches	centimeters	01
feet	meters	02
yards	meters	03
miles	kilometers	04
miles	nautical miles	05
acres	square feet	06
fluid ounces	cubic centimeters	07
fluid ounces	liters	08
gallons	liters	09
ounces	grams	10
pounds	kilograms	11
short ton	metric ton	12
BTU	calories – *gram*	13
degrees	grads	14
degrees	radians	15
°Fahrenheit	°Centigrade	16
deg. min. sec.	decimal degrees	17
polar	rectangular	18
voltage ratio	decibels	19

For example, convert 5 yards to meters. To do so we enter 5 [2nd] 03 and read 4.572 as the result. On the other hand to convert 120 kilometers to miles, we enter 120 [INV] [2nd] 04. The result is 74.56454307.

You can use these codes to convert square units of one system to square units of a second system. For example, to convert 1520 square yards to square meters:

Enter	Press	Display
1520	[2nd] 03 [2nd] 03	1270.913587

In other words, we go through the conversion process twice, effectively multiplying by the conversion constant squared.

In a similar fashion we can convert cubic units of one system to cubic units of another system — convert three times. For example to convert 27 cubic meters to cubic feet:

Enter	Press	Display
27	[INV] [2nd] 02 [INV] [2nd] 02 [INV] [2nd] 02	953.4960015

Notice that each conversion code is a two digit number and leading zeros *must* be entered.

Degrees – Minutes – Seconds/ Decimal Degrees Conversions (Code 17)

Before using this conversion always press [2nd] [Fix Pt] 5, 6, 7 or 8. The format for entering degrees, minutes and seconds is dd.mmss. This notation prescribes the order in which the angle is entered. First enter the number of degrees followed by a *decimal point*. Next enter the *two* digit number for the minutes followed by the digits for seconds and decimal fractions of seconds.

dd.mmss [2nd] **17** – Converts degrees, minutes and seconds to decimal degrees.

n [INV] [2nd] **17** – Converts n number of decimal degrees to degrees, minutes and seconds.

For example, to convert 235°15′30.5″ to decimal degrees we first put the calculator into fixed-point mode. We can use any selection from 5 to 8. In this case, since we have tenths of a second, we use 6.

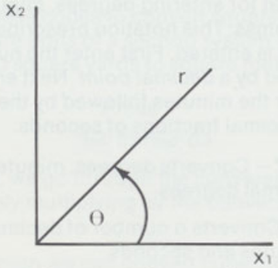
Enter	Press	Display	Comments
	[2nd] [Fix Pt] 6	0.000000	Fix Pt 6 selected
235.15305	[2nd] 17	235.258472	Decimal degrees

Keep the same fixed point accuracy and convert 27.5685 decimal degrees to degrees, minutes and seconds.

Enter	Press	Display	Comments
27.5685	[INV] [2nd] 17	27.340660	The display should be interpreted as 27°34′6.6″

Polar/Rectangular Conversions (Code 18)

The reference system used for polar/rectangular conversions is as shown:



Before beginning the conversion routine set the D/R switch to the angular units desired for both entry and retrieval. Perform the desired coordinate transformation as follows:

r [x:y] Θ [2nd] **18** – Converts polar to rectangular coordinates and displays x_2 coordinate. Press [x:y] to display x_1 coordinate.

x_1 [x:y] **x_2** [INV] [2nd] **18** – Converts rectangular to polar coordinates and displays polar angles Θ.

Press [x:y] to display r coordinate.

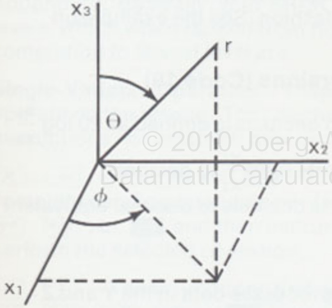
For example, to convert (5, 30°) in polar coordinates to rectangular and then reconvert giving the result in radians:

Angle:Deg

Enter	Press	Display	Comments
5	[x:y]		
30	[2nd] 18	2.5	Value of x_2
	[x:y]	4.330127019	Value of x_1
Angle:Rad			
	[x:y] [INV] [2nd] 18	.5235987756	Θ in radians
	[x:y]	5.	radius

Notice that $\boxed{x:y}$ exchanges the contents of the X and Y registers. As the above key sequence dictates, the radius r must be in the Y register and the angle Θ must be in the X register. For the reverse transformation, the x_1 coordinate must be in the Y register and the x_2 coordinate in the X register. You must have these quantities in the registers specified.

With this information, you can use your SR-51 to convert from spherical coordinates (R, Θ, ϕ) to rectangular coordinates (x_1, x_2, x_3) in a straightforward manner. To convert spherical coordinates to rectangular coordinates, we use the following figure and definitions:



$x_1 = r \sin \Theta \cos \phi$

$x_2 = r \sin \Theta \sin \phi$

$x_3 = r \cos \Theta$

If we first enter r and Θ in the X and Y registers and then convert, we are returned the values r sin Θ which is displayed and x_3 which is in the Y register. Now put the value r sin Θ in the Y register by pressing $\boxed{x:y}$, store x_3 and enter ϕ. Use the conversion routine again. The number displayed after the conversion is x_2 while the number in the Y register is x_1.

For example, let $r = 5$, $\Theta = 30°$, $\phi = 60°$ be the spherical coordinates of a point.

To find the rectangular coordinates:

Solution: Angle:Deg

Enter	Press	Display	Comments
5	x:y		Perform first conversion.
30	2nd 18 x:y STO 1	4.330127019	Store x_3 in memory 1 and places r sin Θ into the Y register.
60	2nd 18	2.165063509	Enter ϕ. Convert second time and display x_2.
	x:y	1.25	Display x_1 Value

Conversions from rectangular to spherical coordinates are handled in a similar fashion. See the example on page 84.

Ratio/Decibel Conversions (Code 19)

The ratio $\frac{x_1}{x_2}$ expressed in decibels is defined as 20 log $\frac{x_1}{x_2}$.

x_1 x:y x_2 2nd 19—Converts ratio $\frac{x_1}{x_2}$ to decibels.

dB INV 2nd 19—Converts decibels to decimal equivalent of a ratio $\frac{x_1}{x_2}$.

Because conversion 18 processes data in the Y and Z registers and conversion 19 processes data in the Y register, any mathematical expression in these registers will be erased. Press C prior to starting new problem.

For example, suppose we have a 35 to 6 ratio to convert to decibels.

Enter	Press	Display	Comments
35	x:y		
6	2nd 19	15.31833588	Ratio in dB

Now to convert 8 dB to the decimal equivalent of a ratio:

Enter	Press	Display	Comments
8	INV 2nd 19	2.511886432	Decimal Ratio

SECTION III

COMPLEX CALCULATIONS

HIERARCHY

Calculator hierarchy determines the precedence (order of completion) of each calculator function. When functions are used individually, hierarchy is of little consequence. However, when functions are used collectively in the solution of an algebraic equation the order of completion is important and can save you many unnecessary key strokes. Your SR-51 uses a sum-of-products precedence which is fully explained in Appendix A. Basically, your SR-51 has four precedence levels, which when ranked from highest level of completion to lowest level are:

Single-Variable Operators – These functions only operate on the display. They neither complete previous instructions nor establish new ones.

[×], [÷], [y^x], [$\sqrt[x]{y}$] and [Δ%] – These functions first complete pending instructions of like kind, i.e., [×], [÷], [y^x], [$\sqrt[x]{y}$], or [Δ%] and then instruct the calculator to perform the selected operation.

[+], [−] – These functions first complete any pending [×], [÷], [y^x], [$\sqrt[x]{y}$] or [Δ%] instructions, then complete any pending [+] or [−] instructions. Finally, they instruct the calculator to perform the selected operation.

[=] – This key first completes any pending [×], [÷], [y^x], [$\sqrt[x]{y}$] or [Δ%] instruction, then completes any pending [+] or [−] instruction. All instructions are completed and the final result is displayed.

Now examine the features of your calculator's hierarchy by working through a few problems in detail.

Example: Cos $(2 \times 30°) = 0.5$

Angle:Deg

Enter	Press	Display	Commands
2	×	2.	Perform Multiplication
30	=	60.	Complete Multiplication
	cos	0.5	Both instructs and completes cosine function

The multiplication key instructs the calculator to perform a multiplication operation. The cosine key (a single-variable operator) does not complete a multiplication instruction as can be seen by referring to the precedence list on page 43. Therefore, before pressing the cos key, we need to complete the multiplication operation. We do this by pressing the = key.

Example: ln $(5 + 10) = 2.708050201$

Enter	Press	Display	Commands
5	+	5.	Perform Addition
10	=	15.	Complete Addition
	lnx	2.708050201	Both instructs and completes logarithm function

The only difference, from a precedence level view point, between this problem and the preceding one is that the addition operation is two levels removed from the single variable operator (lnx) while the multiplication is only one. Nevertheless, the = key must be pressed for the same reason.

Example: $[(2 \times 3) + 4] \times 5 = 50$

Notice that it is necessary to complete the expressions within parentheses just as you would do to solve the problem manually.

Enter	Press	Display	Commands
2	[×]	2.	Perform Multiplication
3	[+]	6.	Complete Multiplication Perform Addition
4	[=]	10.	Complete Addition
	[×]	10.	Perform Multiplication
5	[=]	50.	Complete Multiplication

We proceed with the multiplication of 2 and 3 just as we would for a simple multiplication problem. We observe that the problem as stated seeks to add 4 to the quantity (2×3).Referring to the list of precedence levels previously identified, it is apparent that pressing [+] will first complete the multiplication instruction pending in the calculator. (It effectively performs the first closed parenthesis). However, when we try to perform the second multiplication, we immediately notice that the calculator has a pending addition which the multiplication key *cannot* complete. We, therefore, press the [=] key because it can complete the addition routine without setting up any additional instructions. The remainder of the problem is straightforward.

After working through these case problems, two general rules of hierarchy surface:

1. Whenever a parenthetic bracket contains an operator (calculator function) which is followed by a lower level operator, the second operator will essentially perform an equals instruction (completing the bracket) and establish the appropriate instruction. In this case, you *need not* press the [=] key.

2. Whenever a parenthetic bracket contains an operator which is followed by a higher level operator, the second operator will only set up the appropriate instruction. You *must* press the [=] key prior to pressing the function key in order to complete the bracket.

With a little practice, the calculator hierarchy becomes evident and the use of the [=] key becomes automatic.

SUM OF PRODUCTS

The SR-51 has been designed to calculate sum of products and similar problems in a straightforward manner. Included in this category are sum or difference of products, quotients, powers and roots. Calculations such as these are the most common applications of a calculator memory. In the SR-51, a separate register (the Z register) has been dedicated to this use. This permits computation of this type of problem without use of special memory keys.

Example: $(2 \times 3) + (4 \times 5) = 26$

Enter	Press	Display
2	×	2.
3	+	6.
4	×	4.
5	=	26.

Example: ½ − ¾ = −0.25

Enter	Press	Display
1	÷	1.
2	−	0.5
3	÷	3.
4	=	−0.25

Example: $2^5 - 2^3 = 24$

Enter	Press	Display
2	y^x	2.
5	−	32.
2	y^x	2.
3	=	24.

Example: $\sqrt[3]{8} + \sqrt[4]{625} = 7$

Enter	Press	Display
8	$\boxed{\sqrt[x]{y}}$	8.
3	$\boxed{+}$	2.
625	$\boxed{\sqrt[x]{y}}$	625.
4	$\boxed{=}$	7.

All of the single-variable functions on the SR-51 (see page 16) operate only on the displayed quantity; they do not complete any prior instruction. Thus, the sum-of-products capability can be extended to these functions.

Example: $\sin 30 \cos 60 + \cos 30 \sin 60 = 1$

Angle: Deg.

Enter	Press	Display
30	$\boxed{\sin}$ $\boxed{\times}$	0.5
60	$\boxed{\cos}$ $\boxed{+}$	0.25
30	$\boxed{\cos}$ $\boxed{\times}$	.8660254038
60	$\boxed{\sin}$ $\boxed{=}$	1.

Example: $\dfrac{2 \times 3}{4} + \dfrac{2^3 \times 4}{5} + \dfrac{\sqrt[4]{81} \times 5}{10} = 9.4$

Enter	Press	Display
2	×	2.
3	÷	6.
4	+	1.5
2	y^x	2.
3	×	8.
4	÷	32.
5	+	7.9
81	$\sqrt[x]{y}$	81.
4	×	3.
5	÷	15.
10	=	9.4

PRODUCT OF SUMS

As mentioned previously, the SR-51 was designed to facilitate calculation of the frequently encountered sum-of-products type of problem. As a result, solving the much rarer product of sums is not as direct and requires use of the memory. However, your SR-51 has a PROD key which will facilitate this operation.

Example: $(2 + 3) \times (4 + 5) \times (3 + 4) = 315$

Enter	Press	Display
2	+	2.
3	= STO 1	5.
4	+	4.
5	= 2nd PROD 1	9.
3	+	3.
4	= × RCL 1 =	315.

SECTION IV
SAMPLE PROBLEMS

In the previous sections, you have seen a summary of the basic functions of the SR-51. In this section, we would like to demonstrate the variety of disciplines and decision situations in which your SR-51 can prove invaluable.

BUSINESS AND FINANCE

Depreciation

A salesman buys a car for $4000 that depreciates 30% per year. What is its value at the end of each year for the first six years?

The value V_n at the end of the n^{th} year is calculated using

$$V_n = V_{n-1} - .3V_{n-1} = .7V_{n-1}$$

or in terms of the initial value V_0

$$V_n = (.7)^n V_0$$

Thus, .7 can be used as a constant multiplier. Consequently, one way to obtain a solution to this problem is to use the constant mode feature. In constant mode, when no number is entered prior to pressing [=] the calculator automatically assumes that the number in the display is the entry. To calculate the desired results, we proceed as follows:

Enter	Press	Display	Comments
.7	× 2nd CONST	0.7	Enter constant
4000	=	2800.	First Year
	=	1960.	Second Year
	=	1372.	Third Year
	2nd Fix Pt 2	1372.00	Shift into fixed point 2 with two digits to the right of decimal point.
	=	960.40	Fourth Year
	=	672.28	Fifth Year
	=	470.60	Sixth Year

Notice that when we desired to change to fixed point mode we did so simply by pressing 2nd Fix Pt 2. We were able to do this during the calculation.

Interest

A man wishes to purchase a new car. He finds that he can finance this car over a 36 month period for .8% interest per month on the unpaid balance. With $1000 in savings at 6% interest compounded quarterly, he wishes to decide whether to retain his savings or use it to finance the car.

Part 1
Now the interest saved if the $1000 is used to buy the car is calculated by using the formula:

$$I = nP\left[\frac{1-(1+i)^{-n}}{i}\right]^{-1} - P$$

Where n is the number of payments,

i is the interest per period,

P is the principal or present value.

Thus,

$$I = 36 \times 1000 \left[\frac{1-(1.008)^{-36}}{.008}\right]^{-1} - 1000 = \$154.87$$

The suggested calculator solution is as follows:

Enter	Press	Display	Comments
1	[−]	1.	
1.008	[y^x]	1.008	1 + i
36	[+/−] [=] [÷]	0.249378769	
.008	[=] [1/x] [×]	.0320797156	
36000		36000	36 × 1000
	[−]	1154.869763	
1000	[=] [2nd] [Fix Pt] 2	154.87	Round off to 2 digits to the right of decimal point
	[STO] 1	154.87	

Part II
By keeping his money in the bank compounding quarterly for the same period of time, the total accrued interest would be determined by the following formula:

$$I = P(1+i)^n - P$$

where, i = interest per period or $\frac{.06}{4}$

n = number of periods or 12

Thus

$$I = 1000\left(1+\frac{.06}{4}\right)^{12} - 1000 = \$195.62$$

To solve using the calculator, proceed as follows:

Enter	Press	Display	Comments
1	[+]	1.00	Calculator still in fixed point
.06	[÷]	0.06	
4	[=] [y^x]	1.02	
12	[−]	1.20	
1	[=] [×]	0.20	
1000	[=]	195.62	Amount of interest on $1000

Part III
These results state that the interest accrued is greater than the interest paid on a comparable loan. By pressing [−] [RCL] [1] [=] , he sees that he will gain $40.75 by leaving his savings in the bank. Tax considerations have been neglected.
There are obviously many other problems that can be solved in a similar fashion. The following is intended only as a partial listing.

1. Amortization on a debt
2. Construction of a sinking fund
3. Accumulated interest between two points in time
4. Remaining principal

Appendix E provides a glossary of equations used for solving financial and economic problems. Several examples are provided for reference.

Remaining Balance

What is the remaining balance after 10 years on a 20 year mortgage of $30,000 if the interest rate is 12% per year and monthly payments are $330.33 per month?

From Appendix E, the formula applicable to this problem is:

$$\mathrm{Bal}_K = \mathrm{PMT}\left[\frac{1-(1+i)^{K-n}}{i}\right]$$

where, n = total number of periods

PMT = amount of payment per period

i = interest per period n expressed as a decimal

K = current period

Bal_K = Balance after Kth payment

Solution: K = 10 × 12 = 120

i = 1% per month = 0.01

PMT = 330.33

n = 20 × 12 = 240

Enter	Press	Display	Comments
120	[−]	120.	
240	[=] [STO] 1	−120.	
1	[+]	1.	
0.01	[=] [y^x] [RCL] 1 [=]	−.3029947797	
	[+/−] [+]	−.3029947797	
1	[=] [÷]	.6970052203	
0.01	[×]	69.70052203	
330.33	[2nd] [Fix Pt] 2 [=]	23024.17	Change to Fix Point 2. Remaining Balance

Interest Paid

What is the amount of interest paid on the loan in the preceding problem after 10 years? From Appendix E, the applicable formula for this problem is:

$$Int_K = K\,(PMT) - (PV - Bal_K)$$

where as before:

$K = 120$

$PMT = \$330.33$

$PV = \$30{,}000$

$Bal_K = \$23{,}024.17$

The calculator solution is:

Enter	Press	Display	Comments
30000	[–]	30000.00	
23,024.17	[=] [+/–] [+]	– 6975.83	
120	[×]	120.00	
330.33	[=]	32663.77	Calculator remains in fix point 2 until [2nd] [CA] is pressed.

The interest paid to date is $32,663.77.

Compounded Amounts

At 6% simple interest, how many years will it take $2200 to grow to $10,000. From Appendix E we take the formula:

$$n = \frac{\ln(FV/PV)}{\ln(1+i)}$$

PV = present value = $2200

FV = future value = $10,000

i = interest/period = 6%

n = number of periods

Enter	Press	Display	Comments
10000	[÷]	10000.	
2200	[=] [lnx] [÷]	1.514127733	
1.06	[2nd] [Fix Pt] 1 [lnx] [=]	26.0	Fix decimal point at one place

Answer = 26 years

Annuity

If $100 is deposited each month at 6% simple interest, how much money is accrued at the end of 10 years?

From Appendix E we find:

$$FV = PMT \left[\frac{((1+i)^n - 1)}{i}\right]$$

$i = .06/12 = .005$ = interest per month

$n = 10 \times 12 = 120$ = total number of periods

$$FV = 100 \left[\frac{(1.005)^{120} - 1}{.005}\right]$$

$$= \$16{,}387.94$$

Enter	Press	Display	Comments
1.005	[y^x]	1.005	
120.	[−]	1.819396734	
1	[=] [÷]	.8193967341	
.005	[×]	163.8793468	
100	[2nd] [Fix Pt] 2	100.00	Set decimal point to two places
	[=]	16387.93	

Payment Schedule

If a man arranges a five year $6,000 loan at 9.75% annual interest, what are the monthly payments?

From Appendix E we find:

$$PMT = PV \left[\frac{i}{1-(1+i)^{-n}}\right]$$

$$PV = \$6000$$

$$i = .0975/12 = .008125$$

$$n = 12 \times 5 = 60$$

$$PMT = 6000 \left[\frac{.0975/12}{1-\left(1+\frac{.0975}{12}\right)^{-60}}\right]$$

$$= \$126.75 \text{ per month}$$

Enter	Press	Display	Comments
	2nd Fix Pt 2	0.00	Fix decimal point at two places
.0975	÷	0.10	
12	+ STO 1	0.01	Store interest per month in memory 1
1	= y^x	1.01	
60	+/− =	0.62	
	+/− +	−0.62	
1	= 1/x ×	2.60	
	RCL 1 ×	0.02	Recall interest per period to use as a multiplier
6000	=	126.75	

Trend Analysis

The linear regression feature is extremely useful in predicting trends. For example, over a five year period a certain company reported the following earnings per share. What is the predicted earnings per share for the next five years?

1st five years (known)	2nd five years (predicted)
1. 1.52	6. 3.53
2. 1.35	7. 4.03
3. 1.53	8. 4.52
4. 2.17	9. 5.02
5. 3.60	10. 5.52

What is the expected percent growth from the 9th to the 10th year?

Part I. Because we are performing a trend analysis only the earnings per share need to be entered in sequence. Here is the calculator procedure:

Enter	Press	Display	Comments
	[2nd] [CA]	0	All registers must be cleared at the start of the problem
1.52	[2nd] [y]	1	The 1 indicates data point 1 is entered
1.35	[2nd] [y]	2.	
1.53	[2nd] [y]	3.	
2.17	[2nd] [y]	4.	
3.60	[2nd] [y]	5.	

This completes the data entry. To find the extrapolated values enter the year and ask for the corresponding y' value. That is:

Enter	Press	Display	Comments
6	2nd y'	3.528	6th year
7	2nd y'	4.026	7th year
8	2nd y'	4.524	8th year
9	2nd y'	5.022	9th year
10	2nd y'	5.52	10th year

Part II. The predicted percent change from year nine to year ten is defined as

$$\Delta\% = \frac{y'_{10} - y'_9}{y'_9} \times 100$$

Where y'_{10} = Earnings per share for year 10

y'_9 = Earnings per share for year 9

The calculator solution is as follows:

Enter	Press	Display	Comment
	2nd CA	0	Clear regression routine
5.022	2nd Δ%	5.022	
5.52	2nd Fix Pt 2 =	9.92	9.92 percent increase

The procedures employed in this example apply to many other areas. For example, we could use the steps described above to perform sales forecast or market analysis.

PHYSICAL SCIENCE

Regression Analysis

As another example of regression analysis suppose that two different kinds of measurement were made on the same population with the following results:

x	20.4	19.7	21.8	20.1	20.7
y	9.2	8.9	11.4	9.4	10.3

What is the equation of the regression line that best fits these points?

We wish to find the following expression

$$y = mx + b$$

Where m = slope and b = y intercept. First we enter the data into the calculator:

Enter	Press	Display
	2nd CA	
20.4	2nd x	20.4
9.2	2nd y	1.
.	.	.
.	.	.
.	.	.
.	.	.
20.7	2nd x	20.7
10.3	2nd y	5.

The slope and y intercept are found as follows:

Press	Display	Comments
2nd SLOPE	1.22906793	Slope
2nd INTCP	−15.40505529	y-intercept

Thus the equation of the regression line is

$$y = -15.41 + 1.23x.$$

Regression techniques such as these find diverse applications in many fields. Some potential applications are:

1. Correlating average (or peak) daily temperatures with electrical power consumption to predict expected peak loads.
2. Correlating interest rates with housing sales or business expansion.
3. Correlating a measuring system indications with known inputs.

An important parameter in regression problems is the correlation coefficient or measure of fit of the correlated variables. For a discussion of this, refer to Appendix B.

Half-life of a Radioactive Element

The use of linear regression may be extended to the computation of half-lives using sample data. If we are provided with the following data concerning a sample of radioactive material, what is its half-life?

Quantity (grams)	1.0000	.9747	.8795	.7736	.2770	.0768
Time (days)	0	10	50	100	500	1000

We observe the basic equation $N = N_0 e^{-\lambda t}$ implies that $\ln N = \ln N_0 - \lambda t$.

Thus, we can use the data supplied to perform a semi-logarithmic linear regression to find the half-life. We enter time as the linear x-value and the natural logarithm of the quantity of material as the y-value. The data is entered into the calculator as follows:

Enter	Press	Display
	2nd CA	0
	2nd x	0.
1.	lnx 2nd y	1.
	.	
	.	
	.	
1000	2nd x	1000.
.0768	lnx 2nd y	6.

The half-life is defined as that point where $N_{1/2} = 0.5N_0$. We observe that N_0 is the value of N where $t = 0$. In this particular case $N_0 = 1.0$ and $N_{1/2} = .5$. To calculate the half-life , proceed as follows:

Enter	Press	Display
0.5	lnx 2nd x'	270.0346454

We interpret this as a half-life of 270 days.

As we saw in the previous example, the SR-51 can handle certain nonlinear estimation problems. The idea is to make the nonlinear problem linear by a transformation of variables.

Diffused Junction Characterization

Let us suppose that we are given the following depth versus concentration data.

x (microns)	y (atoms/cm^3)
.50	2.3×10^{19}
1.00	1.1×10^{19}
1.88	2.8×10^{18}
3.00	5.0×10^{18}
4.10	8.6×10^{17}

We know that this data is described by an equation of the form

$$C = C_0 e^{mx}$$

As before, take the natural logarithm of both sides.

$$\ln C = \ln C_0 + mx$$

Now identify y with ln C and ln C_0 with b of the standard form. In this example, we show some new items. First, we can use the fixed-point feature and we can use scientific notation.

Enter	Press	Display	Comments
	[2nd] [CA] [2nd] [Fix Pt] 3	0.000	
	[2nd] [CM]	0.000	
.5	[2nd] [x]	0.500	Enter first data point
2.3	[EE]	2.3 00	
19	[lnx] [2nd] [y]	1.000 00	
1	[2nd] [x]	1.000 00	Enter second data point
1.1	[EE]	1.1 00	
19	[lnx] [2nd] [y]	2.000 00	
.	.	.	Enter third and fourth data points
4.1	[2nd] [x]	4.100 00	Enter fifth data point
8.6	[EE]	8.6 00	
17	[lnx] [2nd] [y]	5.000 00	
	[2nd] [INTCP]	4.468 01	Value of ln C_0
	[eˣ] *	2.534 19	Value of C_0
	[2nd] [SLOPE]	−7.768 −01	Value of m

*In order to find C_0, we have made use of the inverse function relationship of the natural logarithm and the exponential function. For a discussion of inverse functions see Appendix C.

Therefore, $\ln C_0 = 4.468 \times 10 = 44.68$, $C_0 = 2.534 \times 10^{19}$ atoms/cm^3. Also, the slope $m = -7.768 \times 10^{-1} = -.7768$. From this we can see that the concentration at any depth is given by

$$C = 2.534 \times 10^{19} \times e^{-.7768x}$$

STATISTICAL APPLICATIONS

Permutation

Suppose that we have 7 distinguishable items and we wish to determine the maximum number of permutations of any 3 items. We use the expression

$$\text{Perm.} = \frac{n!}{(n-r)!} = \frac{7!}{(7-3)!}$$

To solve on the calculator:

Enter	Press	Display
7	x:y	
3	PRM	210.

Combinations

What are the maximum possible combinations of 8 items taken 5 at a time. We use the relationship

$$C\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{P\binom{n}{r}}{r!} = \frac{P\binom{8}{5}}{5!} = 56$$

To solve on the calculator

Enter	Press	Display
8	x:y	
5	PRM ÷	6720.
5	2nd x! =	56.

Mean, Variance and Standard Deviation

When calculating these statistical parameters, the user may choose to represent his data in any one of several different methods. One way is to treat each datum separately or as *ungrouped data*. In this approach, the user assigns each datum its own value. He sums each datum and divides by the total number of such datum. From this approach comes the more common expression for the mean:

$$\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

where x_i = value of datum point i

N = total number of data points

Another common method exists and is referred to as *grouped data*. In this approach, the range of data (total range of values of x_i) is divided into intervals. All values of x_i which fall into the same interval are assigned a common value, usually the interval midpoint. From this approach arises another expression for the mean:

$$\overline{X} = \frac{\sum_{i=1}^{A} f_i\ x_i}{\sum_{i=1}^{A} f_i}$$

where A = the number of intervals

f_i = number of datum falling in interval i

x_i = value assigned to datum in interval i

$N = \sum_{i=1}^{A} f_i$ = total number of data points

Refer to the basic expression for mean, variance and standard deviation appearing on page 32. By performing some algebraic manipulation it is possible to show that:

$$\bar{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\text{Var.} = \frac{\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}}{N}$$

$$\text{S. Dev} = \left[\frac{\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}}{N-1}\right]^{1/2}$$

By remembering the assignments to each memory when using the [Σ+] key (see page 32), it is possible to rewrite these expressions as follows:

$$\bar{X} = \frac{M1}{M3}$$

$$\text{Variance} = \frac{M2 - \frac{(M1)^2}{M3}}{M3}$$

$$\text{S. Deviations} = \left[\frac{M2 - \frac{(M1)^2}{M3}}{M3 - 1}\right]^{1/2}$$

Where M1 = Contents of memory 1

M2 = Contents of memory 2

M3 = Contents of memory 3

Your SR-51 is programmed to handle ungrouped data. However, as the second example that follows shows, you can handle grouped data easily by using the algorithm that is developed.

Ungrouped Data: What is the mean and standard deviation of the following six capacitor values, 166.4, 167.0, 166.5, 168.6, 171.4, 167.8, drawn at random from a production lot.

Enter	Press	Display	Comments
	2nd CM		
166.4	Σ+	1.	
.	.	.	
.	.	.	Enter data
.	.	.	
167.8	Σ+	6.	
	2nd MEAN	167.95	Mean value
	2nd S. DEV	1.884409722	Standard deviation

Grouped Data: Here is an example to show how the mean and standard deviation of grouped data can be found. For this example we are provided with a set of observations and the frequency with which each datum is observed.

Observation	82	91	90	85
Frequency	2	3	1	7

The expressions we shall use are:

$$\overline{X} = \frac{\sum_{i=1}^{A} f_i x_i}{\sum_{i=1}^{A} f_i} = \frac{M1}{M3}$$

$$\text{S. Dev.} = \left[\frac{\sum_{i=1}^{A} f_i x_i - \dfrac{\left(\sum_{i=1}^{A} f_i x_i\right)^2}{\sum_{i=1}^{A} f_i}}{\sum_{i=1}^{A} f_i \quad - 1} \right]^{1/2}$$

$$= \left[\frac{M2 - \dfrac{(M1)^2}{M3}}{M3 - 1} \right]^{1/2}$$

The principle is to store data into the three memory registers in exactly the same format that the mean and standard deviation routines already preprogrammed in the calculator use them (see page 32). In memory one we must store $\sum_{i=1}^{A} f_i x_i$, in memory two we must store $\sum_{i=1}^{A} f_i x_i^{\,2}$, and in memory three we must store $\sum_{i=1}^{A} f_i$.

Enter	Press	Display	Memory Contents M1	M2	M3
	2nd CM				
2	SUM 3 ×	2.	0.	0.	2.
82	= SUM 1 ×	164.	164.	0.	2.
82	= SUM 2	13448.	164.	13448.	2.
3	SUM 3 ×	3.	164.	13448.	5.
91	= SUM 1 ×	273.	437.	13448.	5.
91	= SUM 2	24843.	437.	38291.	5.
1	SUM 3 ×	1.	437.	38291.	6.
90	= SUM 1 ×	90.	527.	38291.	6.
90	= SUM 2	8100.	527.	46391.	6.
7	SUM 3 ×	7.	527.	46391.	13.
85	= SUM 1 ×	595.	1122.	46391.	13.
85	= SUM 2	50575.	1122.	96966.	13.
	2nd MEAN	86.30769231	(Mean Value)		
	2nd S. DEV	3.275785286	(Standard Deviation)		

Poisson Distribution

The following is a situation that is handled using probability theory. Suppose that the average number of telephone calls arriving at the switchboard of a small company is 30 calls per hour. Some decisions have to be made as to the adequacy of the system as it now is to handle incoming calls promptly. Specifically:

1. What is the probability that no calls will arrive in a 3-minute period?
2. What is the probability that more than two calls will arrive in a 3-minute interval?

We shall assume that the number of calls arriving in any time period has a Poisson distribution. With time measured in minutes, 30 calls per hour is .5 calls per minute. Thus, the mean rate of occurrence is .5 per minute.

The Poisson distribution is given by $P(z) = \dfrac{e^{-vt}(vt)^z}{z!}$

Where v = the mean rate of occurrence

t = the time interval in minutes

z = the number of occurrences.

Therefore, the probability of no calls is determined as:

$$P(0) = \frac{e^{-(.5)(3)}[(.5)(3)]^0}{0!} = e^{-(.5)(3)} = 0.223$$

To solve on the calculator

Enter	Press	Display	Comments
	[2nd] [Fix Pt] 3		Round to 3 decimal places
.5	[×]	.500	
3	[=] [STO] 3 [+/−] [e^x]	0.223	vt is stored in memory 3
	[STO] 1 [STO] 2	0.223	P(0) is stored in memories 1 and 2

We interpret this to mean that the probability of no calls arriving in a three minute interval is 22.3%.

Now the probability of more than 2 calls in a 3-minute interval is:

$$P(z > 2) = 1 - P(0) - P(1) - P(2)$$

$$P(z > 2) = 1 - e^{-(.5)(3)} - \frac{e^{-(.5)(3)}(.5)(3)}{1!} - \frac{e^{-(.5)(3)}[(.5)(3)]^2}{2!} = 0.191$$

We may draw upon a feature of the Poisson distribution, i.e.,

$$P(z = n) = \frac{(P(z = n-1))\ vt}{n}$$

Recalling that vt is stored in memory 3 and P(0) is in memories 1 and 2 we may use the calculator to solve the problem as follows:

Enter	Press	Display	Comments
	RCL 1 × RCL 3 =	0.335	Calculate P(1)
	SUM 2 × RCL 3 ÷	0.502	Add to P(0)
2	= SUM 2	0.251	Calculate (P(2), add to P(0) + P(1) in memory 2
1	− RCL 2 =	0.191	Answer

Product Reliability

A design engineer wishes to predict the reliability of his system. In modeling this product, he notes that any component failure will cause a system failure, i.e., there is no back up. Solution of the system differential equation gives an exponential life curve. Suppose there are two categories of components and n_i ($i = 1, 2$) components of each category. Each component has a mean life defined by $1/\lambda_i$.

Then the probability that component i continues to operate at some time is given by

$$P(t) = e^{-\lambda_i t}$$

Because the operation or failure of each component is an independent event, the probability that his system continues to operate at time t is

$$P(t) = (e^{-\lambda_1 t})^{n_1}\ (e^{-\lambda_2 t})^{n_2}$$

For this example, suppose the data is as follows:

$n_1 = 2,\ 1/\lambda_1 = 100{,}000$ hrs

$n_2 = 3,\ 1/\lambda_2 = 1{,}000{,}000$ hrs

What is the probability that the unit will still be operating at 200 hours? At 400 hours?

Procedure:

Enter	Press	Display	Comments
100,000	[1/x] [STO] 2 [×]	0.00001	
200	[=] [+/−] [e^x] [y^x]	.9980019987	
2	[=] [STO] 1	.9960079893	Component 1 for t = 200 hours
1,000,000	[1/x] [STO] 3 [×]	0.000001	
200	[=] [+/−] [e^x] [y^x]	0.99980002	
3	[=] [2nd] [PROD] 1	0.99940018	Component 2 for t = 200 hours
	[RCL] 2 [×]	0.00001	
400	[=] [+/−] [e^x] [y^x]	.9960079893	
2	[=] [STO] 2	.9920319148	Component 1 for t = 400 hours
	[RCL] 3 [×]	0.000001	
400	[=] [+/−] [e^x] [y^x]	0.99960008	
3	[=] [2nd] [PROD] 2	.9988007197	Component 2 for t = 400 hours
	[2nd] [Fix Pt] 3		
	[RCL] 1	0.995	99.5% continue operation at t = 200 hours
	[RCL] 2	0.991	99.1% continue operation at t = 400 hours.

Return Rate

Suppose that the design engineer in the previous problem wishes to determine the return rate for the first year of operation. Marketing projections indicate that 60% of the units will be operated for an average of 200 hours per year and 40% for an average of 400 hours per year. In this case, the return rate is calculated by the following:

$$RR = .6 \times P_f(200) + .4 \times P_f(400)$$

Where $P_f(200)$ = probability of failure at 200 hours
$P_f(400)$ = probability of failure at 400 hours
$P_f(t) = 1 - P(t)$
$P(t)$ = Probability of operating at time t

To solve on the calculator, we use the data already stored in the calculator from the previous problem and proceed as follows:

Enter	Press	Display	Comments
	2nd Fix Pt 4		Fix. Pt. 4
1	− RCL 1 = ×	0.0046	
.6	= STO 3	0.0028	
1	− RCL 2 = ×	0.0092	
.4	+ RCL 3 =	0.0064	0.64% return rate

Quality Assurance

The following is an example of statistical inference. Suppose that a manufacturing line produces steel rods. The process is designed to produce rods with a mean diameter of 0.500 inches. The following data is collected from a random sample:

.500 .492 .508 .496 .493 .497 .507 .505 .496 .498.

To decide if the process needs adjustment, the quality assurance engineer must first calculate the standard error of the mean, then he must calculate the z-score defined as

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

where $\bar{x}$ = the population mean

$\mu_{\bar{x}}$ = the sample mean

$\sigma_{\bar{x}} = \frac{S}{\sqrt{N}}$ = standard error of the mean

S = the computed standard deviation

N = the sample size

Procedure:

Enter	Press	Display	Comments
	2nd CM		
.5	Σ+	1.	
.492	Σ+	2.	
.	.	.	
.	.	.	Enter data
.	.	.	
.498	Σ+	10.	
.5	−	0.5	Enter given population mean
	2nd MEAN = ÷	0.0008	The difference in means
	2nd S. DEV ×	.1410301484	
	RCL 3 √x =	.4459764877	Value of z

Note that memory 3 is used as a counter and contains the value of N (see page 32).

He uses the result obtained above, 0.45, rounded to two places, to find the corresponding area under the standard normal curve. Consulting a table of normal distribution, he finds that the area from $-\infty$ to $+0.45$ is .6736. Thus, there is a 32.6% probability due to chance alone that the sample mean could be .4992 inches or larger. This is enough to justify the decision to leave the process alone.

ENGINEERING

Aerodynamics

An airplane is in a steady coordinated turn. The true airspeed is 175 knots at a 50° bank angle. What is the turn radius in feet and the turn rate in degrees per second.

The equations used are

$$\text{Turn radius} = \frac{V^2}{g \tan \phi}$$

$$\text{Rate of turn} = W = \frac{g \tan \phi}{V}$$

Where V is in ft/s

W is in radians/s

$g = 32.2\ \text{ft/s}^2$

The calculator solution is as follows:

Angle:Deg

Enter	Press	Display	Comments
	C 2nd Fix Pt 2	0.00	Two-place decimal
32.2	×	32.20	Value of g
50	tan = STO 1	38.37	
175	INV 2nd 05 ×	201.39	Naut. miles to miles conversion
5280	= ÷	1063320.21	Miles to feet
3600	= STO 2 x^2 ÷	87241.50	
	RCL 1 =	2273.43	Value of r in feet
	RCL 1 ÷ RCL 2 =	0.13	Value of w in rad/s
	INV 2nd 15	7.44	Value of w in degrees/s

Meter Correction

An electrical engineer wishes to measure the signal level of a voice channel having an equivalent impedance of 150Ω. The power measuring test set that he has can provide termination resistance of either 600Ω or 150Ω but the deflecting meter is only calibrated for 600Ω. What correction factor should he apply to the meter readings when the channel is terminated in 150Ω.

He knows that

$$\text{power ratio dB} = 10 \log \frac{W_1}{1 \text{ mw}}.$$

Where $\frac{V^2}{600} = 1$ mw

when 600Ω termination is used.

$$W_1 = \frac{V^2}{150}$$

Thus, the power ratio, PR,

$$PR = 10 \log \frac{\frac{V^2}{150}}{\frac{V^2}{600}} = 10 \log \frac{600}{150} = 6.02 \text{ dB}$$

Before he uses the calculator, he recalls that the voltage ratio to decibel conversion provided with the calculator is defined as $20 \log \frac{X_1}{X_2}$. Thus he must divide his result by two to obtain the answer he needs.

The calculator solution is as follows:

Enter	Press	Display	Comments
	[2nd] [CA] [2nd] [Fix Pt] 2	0.00	Two-place decimal
600	[x:y]	0.00	
150	[2nd] 19 [÷]	12.04	
2	[=]	6.02	Correction factor in dB

The meter will read 6.02 dB high. He must subtract 6.02 dB from his meter readings.

Cam Problem

What is the minimum base radius of a cycloidal cam having a maximum pressure angle of 25°, a cam angle of 75°, and a rise of 1.25 inches. The line of action passes through the center of the cam.

L = stroke = 1.25 inches

β = Total cam angle = 75°

R = Base radius

α = Pressure angle = 25°

By differentiation and maximizing the rate of change of the radius the following formula can be established:

$$R = \frac{2L}{\beta \tan \alpha} - \frac{L}{2}$$

where β is expressed in radians

Therefore:

$$R = \frac{2\,(1.25)}{\beta \tan 25°} - \frac{1.25}{2}$$

$$R = 3.470706523 \text{ inches}$$

Solution: Angle:Deg

Enter	Press	Display
2	[×]	2.
1.25	[÷]	2.5
75	[2nd] 15 [÷]	1.909859317
25	[tan] [−]	4.095706523
1.25	[÷]	1.25
2	[=]	3.470706523

Torque Problem

What is the torque required to drive the above cam at the maximum pressure angle? The maximum pressure angle occurs at ½ the total rise (radius $= R + \frac{L}{2}$), and against a total work load of 25.7 lbs.

$$\text{Torque} = (W)(\tan x_{mx})(r)$$
$$= 25.7 (\tan 25°)(3.470706523 + 1.25/2)$$
$$= 49.08338445 \text{ in} \cdot \text{lb}$$

Solution: Angle:Deg

Enter	Press	Display
3.470706523	[+]	3.470706523
1.25	[÷]	1.25
2	[=] [×]	4.095706523
25	[tan] [×]	1.909859317
25.7	[=]	49.08338445

Chain Drive Problem (Timing Belts)

With a 1.25″ pitch chain ($P = 1.25″$), a 24 tooth driver ($N_1 = 24$) a gear down ratio of 6 to 1 and a desired center distance of 60 inches ($C_d = 60″$), what is the required pitch length of chain (PL) and the actual center distance (C_A)?

Driver = 24T   Driven = 6(24)T = 144T

$$\text{Pitch Dia} = \frac{P}{\sin\left(\frac{180}{N}\right)}$$

PD = Larger Pitch Dia.

Pd = Smaller PD.

$$PL = 2C \cos \phi + \frac{\pi}{180} (PD\ (90 + \phi) + Pd\ (90 - \phi))$$

Where $\phi = \sin^{-1} \frac{(PD - Pd)}{2\ C_d}$ = A very close approximation.

$$C_A = \frac{1}{2 \cos \phi} \quad PL - \frac{\pi}{180} (PD\ (90 + \phi) + Pd\ (90 - \phi))$$

$$PD = \frac{1.25}{\sin\left(\frac{180}{144}\right)} = 57.3003249'' = 57.300''$$

$$Pd = \frac{1.25}{\sin\left(\frac{180}{24}\right)} = 9.576621969'' = 9.577''$$

$$\phi = \sin^{-1} \frac{(57.3 - 9.577)}{2(60)} = 23.43395251° = 23.434°$$

$90 + \phi = 113.4339525$ $90 - \phi = 66.56604749$

$$PL = 2\ (60) \cos \phi + \frac{\pi}{180} (PD\ (90 + \phi) + Pd(90 - \phi))$$

$$= 234.6711298 \text{ inches}$$

The chain PL must be an even integer number of pitches

$PL = \frac{234.67}{1.25}$ rounded to next highest even number $\times$ 1.25

$PL = 188 \times 1.25 = 235$ inches   Ans #1

$$C_A = \frac{1}{2 \cos \phi} 235 - \frac{\pi}{180} (57.3(90+\phi) + 9.577(90-\phi))$$

$C_A = 60.1792$ inches   Ans #2

Solution: Angle:Deg

Enter	Press	Display	Comments
	2nd Fix Pt 4		
180	÷	180.0000	
144	= sin 1/x ×	45.8403	
1.25	= STO 1	57.3003	PD in M1
180	÷	180.0000	
24	= sin 1/x ×	7.6613	
1.25	= STO 2 +/− +	9.5766	PD in M2
	RCL 1 = ÷	47.7237	
2	÷	23.8619	
60	= INV sin		
	STO 3 +	23.4343	
90	= 2nd PROD 1	113.4343	PD(90 + 0) in M1
	RCL 3 +/− +	−23.4343	
90	= 2nd PROD 2	66.5657	PD(90 − 0) in M2
	RCL 1 + RCL 2		
	= × π ÷	22422.4819	
180	= STO 1 +		
	RCL 3 cos ×	0.9175	(Expressions) in M
60	×	55.0510	
2	= ÷	234.6713	PL
1.25	=	187.7371	Round to next highest even number.
188	×	188.0000	
1.25	= −	235.0000	Ans #1 PL in inches
	RCL 1 = ÷		
	RCL 3 cos ÷	120.3582	
2	=	60.1791	Ans #2 C_λ in inches

Device Parameter Calculation

Suppose we have a p-channel enhancement-mode MOSFET and that we need to calculate the source to drain conductance (g_{sd}) operating in the triode region. The formula for obtaining this result is

$$g_{sd} = \frac{\mu \epsilon_{ox} \epsilon_o W}{t_{ox} L} \cdot \left| V_G - V_T - V_D \right|$$

Where

μ is carrier mobility	$= 190\ cm^2/V-s$
ϵ_{ox} is relative dielectic of the oxide	$= 4$
t_{ox} is thickness of oxide over channel	$= 1000Å$
W is width of channel	$= 1$ mil
L is length of channel	$= 0.2$ mil
V_G is the gate voltage	$= -10$ V
V_T is the threshold voltage	$= -4$ V
V_D is the drain voltage	$= -1$ V
ϵ_o is permittivity of free space	$= 8.85 \times 10^{-14}$F/cm

Now

$$g_{sd} = \frac{(190)(4)(8.85 \times 10^{-14})(1)}{(10^{-5})(0.2)} \left| -10 + 4 + 1 \right|$$

$$= 1.6815 \times 10^{-4} \text{ mhos}$$

Thus the channel resistance is

$$r_d = \frac{1}{g_{sd}} = 5.947 \times 10^3 \text{ ohms.}$$

The following is a suggested calculator routine:

Enter	Press	Display	Comments
.2	EE	0.2 00	
5	+/− 1/x ×	5. 05	
190	×	9.5 07	
4	×	3.8 08	
8.85	EE	8.85 00	
14	+/− = STO 1	3.363 −05	
10	+/− +	−1. 01	
4	+	−6. 00	
1	= +/− × RCL 1 =	1.6815 −04	g_{sd} in mhos
	1/x 2nd Fix Pt 3	5.947 03	r_d in ohms

Now since we have the quantity

$$\beta = \frac{\mu \epsilon_{ox} \epsilon_o W}{t_{ox} L}$$

(some books use $K = \frac{\mu \epsilon_{ox}}{2t_{ox}} \cdot \frac{W}{L}$ as the SAH equation)

stored in memory one, we can compute the transconductance in the triode region and saturation region. In the first case:

$$g_m = \beta |V_D|$$

In the second case

$$g_m = \beta \mid V_G - V_T \mid$$

Solution:

Enter	Press	Display	Comments
	RCL 1 ×	3.363 –05	$V_D = -1$, thus $\lvert V_D \rvert = 1$
1	=	3.363 –05	g_m in mhos in triode region
10	–	1.000 01	$V_G = -10$, $V_T = -4$
4	= × RCL 1 =	2.018 –04	g_m in mhos in saturation region

MATHEMATICS

Vector Addition

Add the following vectors:

$$5 \angle 30^\circ + 10 \angle 45^\circ = r' \angle \Theta'$$

Our solution is to first find the individual x and y components of each vector using the polar rectangular conversion routine. Next we sum both x and y components separately to achieve the resultant X and Y values. The equations used are

$$X = 5 \cos 30^\circ + 10 \cos 45^\circ$$

$$Y = 5 \sin 30^\circ + 10 \sin 45^\circ$$

Finally, we perform a rectantular to polar transformation on the X and Y resultant values to arrive at r′ and Θ′. The equations used are:

$$r' = \sqrt{X^2 + Y^2} = 14.88598612$$

$$\Theta' = \tan^{-1} \frac{Y}{X} = 40.01276527$$

The calculator solution is: Angle:Deg

Enter	Press	Display	Comments
5	[x:y]		Enter radius of first vector.
30	[2nd] 18 [STO] 1	2.5	Enter angle of first vector, complete polar
	[x:y] [STO] 2	4.330127019	rectangular conversion. y stored in M1 and x stored in M2.
10	[x:y]	2.5	Enter radius of second vector.
45	[2nd] 18 [SUM] 1	7.071067812	Enter angle of second vector, complete polar/ rectangular
	[x:y] [SUM] 2	7.071067812	conversion. Sum y components in M1 and x components in M2.
	[RCL] 2 [x:y] [RCL] 1	9.571067812	Resultant X and Y components recalled for rectangular/ polar conversion.
	[INV] [2nd] 18	40.01276527	Angle Θ′ in degrees
	[x:y]	14.88598612	Magnitude r′

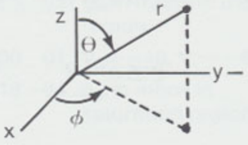
Rectangular/Spherical Coordinate Conversions

To convert (5, 8, 10) from rectangular to spherical coordinates use the following reference system.

Where $r = x^2 + y^2 + z^2$

$$\phi = \tan^{-1} \frac{y}{x}$$

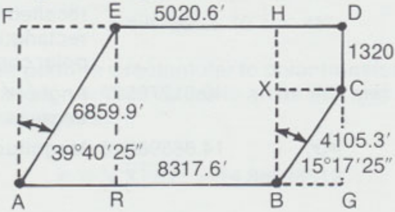
$$\Theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$



To solve on the calculator: Angle:Deg

Enter	Press	Display	Comments
5	x:y		Exter x
8	INV 2nd 18	57.99461679	Enter y; value of ϕ displayed in degrees.
10	x:y INV 2nd 18	43.33171975	Enter z; value of Θ displayed in degrees.
	x:y	13.74772708	Value of r

Area of Irregular Polygons



An investor wishes to purchase the tract of land shown for future development. With land prices at $1200 per acre, how much can he expect to spend? The parts of the figure have been labeled to help you follow the solution.

$$(\text{Total area}) \times (\text{Price/unit area}) = \text{Total Cost}$$

$$\text{Total area} = AGDF - AEF - BGC$$

Here is the calculator procedure: Angle:Deg

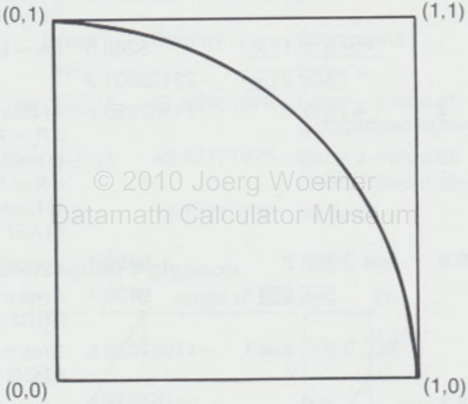
Enter	Press	Display	Comments
6859.9	x:y 2nd Fix Pt 1		Side AE
39.4025	2nd 17	39.6	Converts angle FAE to decimal degrees.
	2nd 18 STO 2	4379.5	Polar/rectangular conversion. Display shows FE = AR
	x:y STO 1 ×	5280.0	FA = ER
	RCL 2 ÷	23123601.2	
2	= STO 3	11567080.6	At this point, AR = FE is in M2, ER = FA in M1, and area of AEF in M3.
5020.6	SUM 2 RCL 2	9400.1	Length of FD
	2nd PROD 1	9400.1	Area of AGDF in M1
	RCL 3 +/− SUM 1	−11567080.6	Area of AEDG in M1
4105.3	x:y	23123601.2	
15.1725	2nd 17 2nd 18 STO 2	1082.6	BG in M2
	x:y × RCL 2 ÷	4287099.9	
2	=	2147510.0	Area of BGC
	+/− 1 RCL 1	35917887.2	Area of AEDCB
	INV 2nd 06 ×	824.6	ft^2 to acres conversion
1200	= 2nd Fix Pt 2	989473.48	Cost for plot

Approximation Methods

The SR-51 can quite effectively aid in the solution of problems which require approximations. Here are just a few of many examples. The first method we shall illustrate is the so-called Monte Carlo method to find the area of a circle.

Monte Carlo Method

We shall assume that a quarter circle lies in the first quadrant as shown below.



Now a point (x, y) is an element of the circle if and only if $x^2 + y^2 \leq 1$. Since the random number generator generates two digit random numbers we shall look for sums of square of pairs of random numbers which are less than 10000. We could of course divide each random number by one hundred. But this scaling introduces superfluous key strokes. Table II shows a typical set of data produced by the following key sequence:

2nd RAN# x^2 + 2nd RAN# x^2 =

Table II

	x	x^2	y	y^2	$x^2 + y^2$	
1	75	5625	5	25	5650	
2	86	7396	51	2601	9997	
3	41	1681	49	2401	4082	
4	99	9801	73	5329	15130	*
5	96	9216	32	1024	10240	*
6	88	7744	58	3364	11108	*
7	78	6084	63	3969	10053	*
8	4	16	0	0	16	
9	98	9604	31	961	10565	*
10	45	2025	79	6241	8266	
11	27	729	58	3364	4093	
12	89	7921	7	49	7970	
13	53	2809	54	2916	5725	
14	61	3721	87	7569	11290	*
15	43	1849	82	6724	8573	
16	70	4900	36	1296	6196	
17	79	6241	6	36	6277	
18	29	841	35	1225	2066	
19	54	2916	9	81	2997	
20	30	900	10	100	1000	

*These are unsuccessful trials.

In 20 trials, therefore, we count 14 successes. We can estimate the probability of landing within the quarter circle as $\frac{14}{20}$ or .7. Monte Carlo theory predicts we should get $\frac{\pi}{4} = .785$ as the probability of finding a point in the quarter circle. Notice that this is also the area of the quarter circle. Our results predict $.7 \times 4 = 2.8$ as the area of a circle. This compares reasonably well with the actual value of π for the area. We can of course increase the accuracy by increasing the number of trials. In fact, out of 100 trials, we obtained $\frac{80}{100}$ or .8 as the probability of hitting within the quarter circle. This gives an area of 3.2 for the whole circle.

Approximating Integrals

The preceding result compares favorably with the following use of Simpson's rule of approximate integration. The area of the quarter circle is given by

$$A = \int_0^1 \sqrt{1 - x^2}\, dx$$

Simpson's rule states that

$A = \frac{1}{3} h[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$; n is even, h is the length of the uniform subdivisions, and y_i is value of the function at each division point, x_i, of the interval of integration.

For comparison, we give two solutions. On the interval [0,1] we pick $h = \frac{1}{2}$, so $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$.

Therefore,

$$A = \int_0^1 \sqrt{1 - x^2}\,dx = \frac{1}{3} \cdot h \left[(y_0 + y_2) + 4(y_1)\right]$$

$$= \frac{1}{3} \cdot \frac{1}{2} \left[\sqrt{1 - 0^2} + \sqrt{1 - 1^2} + 4\sqrt{1 - (\tfrac{1}{2})^2}\right]$$

$$= .7440169359$$

On the calculator we proceed as follows:

Enter	Press	Display	Comments
1	[−]	1.	
.5	[x^2] [=] [$\sqrt{x}$] [×]	.8660254038	
4	[+]	3.464101615	
1	[=] [÷]	4.464101615	
6	[=]	.7440169359	Approximate area A

Accuracy improves considerably if we take four subintervals instead of the two above. Notice that some simple preliminary arithmetic helps in producing an efficient calculator algorithm. The following equation expresses the situation for four subintervals:

$$h = \frac{1}{4},\ x_0 = 0,\ x_1 = \frac{1}{4},\ x_2 = \frac{1}{2},\ x_3 = \frac{3}{4},\ x_4 = 1$$

$$A = \int_0^1 \sqrt{1 - x^2}dx = \frac{1}{3}h\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \frac{1}{3} \cdot \frac{1}{4}\left[\left(\sqrt{1-0^2} + \sqrt{1-1^2}\right) + 4\left(\sqrt{1-.25^2} + \sqrt{1-.75^2}\right)\right.$$

$$\left. + 2\sqrt{1-.5^2}\right]$$

$$= .7708987887$$

Solution:

Enter	Press	Display	Comments
	2nd CM		To clear memories
1	−	1.	
.25	x^2 = $\sqrt{x}$ SUM 1	.9682458366	
1	−	1.	
.75	x^2 = $\sqrt{x}$ SUM 1	.6614378278	
4	2nd PROD 1	4.	Multiplies contents of M1 by 4
1	−	1.	
.5	x^2 = $\sqrt{x}$ ×	.8660254038	
2	= + RCL 1 +	8.250785465	
1	= ÷	9.250785465	
12	=	.7708987887	Approximate area A

Approximating Derivatives

Your SR-51 can also aid in the approximation of derivatives. For example, let's approximate the derivative of $f(x) = \sin x$ at $x_0 = 45°$ or $\frac{\pi}{4}$ radians. Recall that if $f(x) = \sin x$, then $f'(x) = \cos x$. Also,

$$f'(x_0) = \lim_{\Delta x \to 0} \left[\frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} \right]$$

The calculator algorithm for this process is:

1. Convert 45° to radians and store in M1.

2. Add contents of 1 to .0001, take sin and store in M2.

3. Subtract .0001 from contents of M1. Take the sin, change sign and add to contents of M2.

4. Multiply 2 and .0001, take reciprocal and multiply the result times contents of M2.

Calculator solution: Angle:Rad

Enter	Press	Display	Comments
45	2nd 15 STO 1 +	.7853981634	
.0001	= sin STO 2	.7071774883	
	RCL 1 −	.7853981634	
.0001	= sin +/− SUM 2	−0.707036067	
2	×	2.	
.0001	= 1/x ×	5000.	
	RCL 2 =	.7071067815	Value of $f'(\frac{\pi}{4})$
	− RCL 1 cos =	.0000000003	Difference of $f'(\frac{\pi}{½})$ and $\cos\frac{\pi}{4}$

Solution of a Differential Equation

Now suppose we have a differential equation of the form $y' = f(x,y)$, $y(0) = a$. It can be shown that approximate solutions can be obtained by using the following recursive equation: $y_{n+1} = y_n + hf(x_n, y_n)$. Therefore, to solve $y' = x + y$, $y(0) = 0$, $h = .2$, our recursion relation becomes:

$$y_{n+1} = y_n + h(x_n + y_n)$$

Where $x_n = nh$

By inspection, the value of $y_{n+1} = 0$, with $n = 0$. Therefore the calculator solution will begin with $n = 1$ and $h = 0.2$.

Enter	Press	Display	Comments
	[2nd] [Fix Pt] 3		
0	[STO] 1 [+]	0.000	Store y_{n+1} value in M1.
1	[×]	1.000	Enter value of n
.2	[=] [×]	0.200	Value of $(x_n + y_n)$
.2	[+] [RCL] 1 [=]	0.040	Value of y_{n+1}
	[STO] 1 [+]	0.040	Store y_{n+1} as new y_n in M1
2	[×]	2.000	Enter next value of n
.2	[=] [×]	0.440	
.2	[+] [RCL] 1 [=]	0.128	Value of Y_{n+1} to be used as y_n in next sequence

Since the procedure is repetitive, the results of ten calculation sequences are shown in Table III. Also, for comparison of accuracy, Table III shows the actual y_{n+1} value computed with the equation:

$$y = e^{x_n} - x_n - 1$$

Table III

n	x_n	y_n	$y_n + h(x_n + y_n)$	actual y-value
0	0	0	0	0
1	.2	0	.04	.021
2	.4	.04	.128	.092
3	.6	.128	.274	.222
4	.8	.274	.488	.426
5	1.0	.488	.786	.718
6	1.2	.786	1.183	1.12
7	1.4	1.183	1.7	1.655
8	1.6	1.7	2.360	2.353
9	1.8	2.360	3.192	3.250
10	2.0	3.151	4.230	4.389

The accuracy of the above algorithm can be increased by selecting a smaller value of h.

Solution of Algebraic Equations

Using similar iterative techniques we may solve algebraic equations. For example, consider the following equation.

$$f(x) = x^3 + x - 1 = 0$$

It is easily determined using Descartes' rule of signs that this equation has exactly one real positive root. We shall approximate the reat root by noting that we can rewrite the equation as

$$x = \frac{1}{1 + x^2}$$

Thus, we obtain an approximation routine using the form

$$x_{n+1} = \frac{1}{1 + x^2_n}$$

We start our routine in Table IV with x = 0 which is an arbitrary guess. The routine will in general correct itself.

Table IV

n	x_n	x_{n+1}
0	0	1
1	1	.5
2	.5	.8
3	.8	.610
4	.610	.729
5	.729	.653
6	.653	.701
7	.701	.670
8	.670	.690
9	.690	.678
10	.678	.685

Notice in Table IV that each derived x is squared, the result is added to 1 and the reciprocal taken of that sum. For example:

Enter	Press	Display	Comments
	2nd Fix Pt 3		
.653	x^2 +	0.426	
1	= 1/x	0.701	Value of x_{n+1} with n = 6
	x^2 +	0.491	
1	= 1/x	0.670	Value of x_{n+1} with n = 7

To check how near we are to a solution at the tenth step shown in Table IV:

Enter	Press	Display
.685	STO 1 y^x	0.685
3	+ RCL 1 −	1.006
1	=	0.006

Therefore, we are 0.006 away from zero and more iterations will be necessary for more accuracy.

NAVIGATION

In flight planning problems the wind triangle is used. It consists of three vectors and six factors as shown in the following diagram.

Vector	Angle or Heading	Velocity
AB	TC, true course	PGS, predicted ground speed
AC	WD, wind direction	WV, wind velocity
CB	TH, true heading	TAS, true air speed

In preflight plans, the known data are true course, wind direction, wind velocity, and true air speed.

To find the true heading if TC = 40°, WD = 105°, TAS = 120 mph, and WV = 45 mph, it can be shown that:

$$TH = TC - \sin^{-1}\left[\frac{WV \sin (WD - TC)}{TAS}\right]$$

Solution: Angle:Deg

Enter	Press	Display	Comments
	[2nd] [Fix Pt.] 5		
105	[−]	105.00000	
40	[=] [sin] [×]	0.90631	
47	[÷]	42.59647	
120	[=] [INV] [sin] [+/−] [+]	−20.79164	
40	[=] [INV] [2nd] 17	19.12300	Read answer as 19°12′30.0″

To find the predicted ground speed, we use the following expression

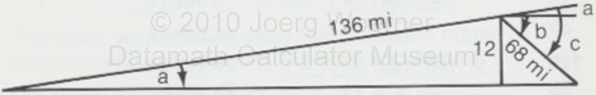
$$PGS = WV \cos (WD - TC) + \sqrt{[WV \cos (WD - TC)]^2 - (WV)^2 + (TAS)^2}$$

$$= 47 \cos (105 - 40) + \sqrt{[47 \cos (105 - 40)]^2 - 47^2 + 120^2}$$

Solution: Angle:Deg

Enter	Press	Display	Comments
	[2nd] [Fix Pt] 2		
105	[−]	105.00	
40	[=] [cos] [×]	0.42	
47	[=] [STO] 1 [x^2] [−]	394.54	
47	[x^2] [+]	−1814.46	
120	[x^2] [=] [$\sqrt{x}$] [+]	112.19	
	[RCL] 1 [=]	132.05	PGS in miles/hour

Finally, we calculate the drift correction assuming that during the flight the plane is 12 miles off course. If the distance flown is 136 miles and the remaining distance is 68 miles, we need to know the angle for a parallel course, the correction angle to converge at destination and the total correction to converge on the destination.



$$a = \sin^{-1} \frac{12}{136}$$

$$b = \sin^{-1} \frac{12}{68}$$

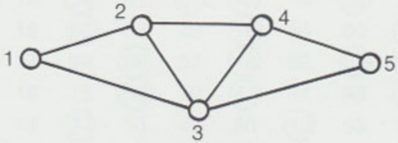
$$\text{total correction } c = a + b$$

Solution: Angle:Deg

Enter	Press	Display	Comments
	2nd Fix Pt. 4		
12	÷	12.0000	
136	= INV sin STO 1	5.0621	Value of a
12	÷	12.0000	
68	= INV sin +	10.1642	
	RCL 1 =	15.2263	Decimal degrees
	INV 2nd 17	15.1334	Read as 15°13′34″ for correction angle c

SIMULATION

The following problem shows how the SR-51 can be used to simulate a situation again using the built in random-number generator. Suppose we have a telephone network with five stations as shown below.



We wish to place a call from 1 to 5 and we wish to obtain some kind of estimate of the probability of completion of the call. There are seven trunks and each trunk has a certain probability of being busy. It is easier, however, to work with its probability of being open.

These probabilities are given as follows:

Trunks	(12)	(13)	(23)	(24)	(34)	(35)	(45)
Prob. of being open	.7	.2	.3	.6	.3	.2	.7
Trunk open if Random # is	00-69	00-19	00-29	00-59	00-29	00-19	00-69
Trunk busy if Random # is	70-99	20-99	30-99	60-99	30-99	20-99	70-99

We use our random-number generator to simulate the situation at any given moment as illustrated in Table V. For each trial we generate one random number for each trunk. For example we press [2nd] [RAN#] for trunk (12). If the random number is any number from 00-69, the trunk is considered open. If the number is 70-99, the trunk is busy. We repeat the process for each trunk, completing a single trial.

Table V

Trunk		(12)	(13)	(23)	(24)	(34)	(35)	(45)	(15)
Prob:		.7	.2	.3	.6	.3	.2	.7	
Case	1	87	83	70	52	61	65	05	
	2	19	31	65	45	59	56	03	open
	3	88	92	45	43	11	97	94	
	4	01	07	32	80	91	37	36	
	5	03	35	94	14	66	48	82	
	6	15	97	90	56	04	16	08	open
	7	34	01	55	58	72	17	03	open
	8	25	36	94	22	30	50	60	open
	9	83	59	57	97	62	78	67	
	10	93	11	95	90	82	59	12	
	11	27	29	20	65	32	04	68	open
	12	77	14	75	51	73	83	06	
	13	57	40	27	90	58	12	24	open
	14	50	51	79	38	73	63	17	open
	15	24	81	07	26	48	03	81	open
	16	69	53	79	18	88	90	36	open
	17	96	08	83	18	86	02	09	open
	18	75	00	70	47	44	98	35	
	19	34	53	49	88	13	53	99	
	20	43	86	98	44	63	37	02	open

Thus we can estimate from Table V that the probability of an open line is 0.55.

Several such simulations as this might provide the basis upon which to make some kind of decision about adding new lines to the existing trunks in this network or adding new trunks. In this case we see that the simulation indicates that we can complete a call from station one to station five 55% of the time. Thus, depending on the importance of completing such a call, we might very well wish to increase line capacity.

With the SR-51 and its random-number generator one can deal with a large number of decision problems that require simulation. In particular, one can simulate items entering and leaving a queue, items being bought in a store and many others.

APPENDIX A
REGISTER LEVEL PROCESSING

To provide additional insight into the data processing of your SR-51, the following discussion is included to show the details of processing at the register level. These registers are electronic elements used to store data while it is being displayed, being processed, or waiting to be processed. Please note that your SR-51 relieves you of the burden of keeping track of the contents of the registers and of assigning functions. With the straightforward approach of algebraic entry and its defined sequence of processing, the calculator automatically controls the register and function assignment.

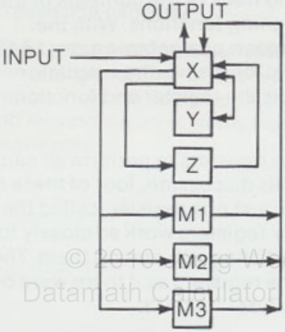
Your SR-51 uses nine registers to perform all calculations. For the purpose of this discussion, four of these registers will be considered as just one register, called the X register. These four registers work so closely together that the user cannot distinguish among them. Therefore, we shall treat the nine registers as if there were only six and label them X, Y, Z, M1, M2, M3.

Upper case letters are used to indicate the register, while lower case letters are used to indicate the contents of each register; x in X register, y in Y register, and z in Z register. Three memory registers, M1, M2 and M3, are used as memory locations and for processing elements in some routines.

The X register is the input register. The value of x is the quantity shown on the display. The value of x is always the operand for single-variable functions; it is processed and returned to X without changing the Y and Z registers. One operand for multiplication or division, and one operand for two-variable functions is always stored in the X register. The Y register is a holding register.

It holds the second operand for multiplication or division and the second operand for two-variable functions. The Z register is the sum-of-products register. Registers M1, M2 and M3 are used as the memory locations for storage, summing in memory and multiplying in memory. They are used for processing mean, variance, and standard deviation and linear regression routines.

The following diagram shows the inter-register data flow.



Before examining the registers, we first group the keys according to the levels on which they operate (see page 43). Excluding the special mean, variance, and standard deviation and linear regression routines, there are four levels of key operations in your SR-51. These are listed in descending order from highest to lowest.

1. The A level consists of sin, cos, tan, sinh, cosh, tanh, and their inverses; also %, lnx, log, e^x, 10^x, x^2, $\sqrt{x}$, $1/x$, $x!$, RAN#; conversions 00-17, and the inverses of conversions 00-17.

2. The B level consists of ×, ÷, y^x, $\sqrt[x]{y}$, Δ%.

3. The C level consists of +, −.

4. The D level consists of =.

The A-level operator acts only on the display and hence just on the X register.

The B-level operator first completes any B-level instruction pending and stores that result in the Y register. It then sets up a new B-level instruction and copies the contents of the Y register into the X register.

The C-level operator first completes any pending B-level instruction, then any pending C-level instruction and stores the result in the Z register. It then sets up a new C-level instruction and copies the contents of the Z register into the X register.

The D-level operator first completes any B-level operation pending then any C-level operation pending. In addition $\boxed{=}$ clears the Z register.

To see this more clearly, consider the following examples:

B followed by B	Enter	X	Y	Z	Comments
a B b B	a	a			B repeats a in Y
	B	a	a		
	b	b	a		
	B	a B b	a B b		B completes previous B instruction
B followed by C					
a B b C	a	a			
	B	a	a		
	b	b	a		
	C	a B b	a	a B b	C completes B instruction and repeats the result in Z.
C followed by B					
a C b B	a	a			
	C	a		a	C repeats a in Z
	b	b		a	
	B	b	b	a	Note that two operations are pending in the calculator.
C followed by C					
a C b C	a	a			
	C	a		a	
	b	b		a	
	C	a C b		a C b	

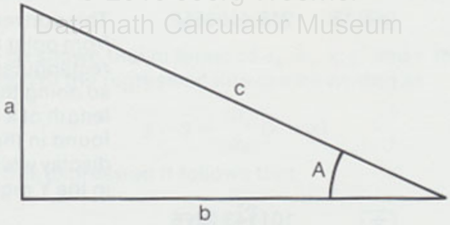
B followed by D	Enter	X	Y	Z	Comments
a B b D	a	a			
	B	a	a		
	b	b	a		
	D	a B b	a		D completes B, puts the result in X and leaves a in Y
C followed by D					
a C b D	a	a			
	C	a		a	
	b	b		a	
	D	a C b			D completes C, puts the result in X and clears Z
B followed by C followed by D					
a B b C c D	a	a			
	B	a	a		
	b	b	a		
	C	a B b	a	a B b	
	c	c	a	a B b	
	D	a B b C c D	a		
C followed by B followed by D					
a C b B c D	a	a			
	C	a		a	Completes the B operation first then completes the pending C operation. For example, $2+3\times4=2+12=14$, multiplication is completed before addition.
	b	b		a	
	B	b	b	a	
	c	c	b	a	
	D	a C b B c D	b		

Memory registers M1, M2 and M3 are not affected by A, B, C or D level operators. All memory related keys transfer data between memory registers and the X register. Because they do not interact with Y or Z registers, they may be used at any point in a calculation without destroying a mathematical expression.

When you choose to use the mean, variance and standard deviation routine or the linear regression routine, the role of the memory registers changes to that of processing registers. Because during these routines data is processed in registers M1, M2 and M3, they may not be used as memories.

Conversion routines 18 and 19, PRM and CONST should be viewed as isolated calculations performed independently of arithmetic expressions (B and C level operators).

Knowledge of register-level processing suggests a way to calculate the area of a right triangle, given the base angle and the length of the hypotenuse.



For example: let C = 512, A = 25° 16′ 6″

Solution: Angle:Deg

Enter	Press	Display	Comments
	2nd CA ×	0.	
512		512	
	x:y	0.	This puts 512 in Y register, multiplication is still recorded.
25.1506		25.1506	The angle in degrees, minutes and seconds.
	2nd 17	2.525166666 01	Converts angle to decimal degrees. Only display is affected.
			A is in the display, r is in the Y register.
	2nd 18	218.4166652	This converts from polar to rectangular. In so doing the length of a is found in the display while b is in the Y register.
	÷	101143.2276	
2	=	51034.68849	Displayed number is the area.

APPENDIX B
SIMPLE LINEAR CORRELATION

In addition to the regression line analysis given earlier in this manual, there is another way to measure the degree of association between the x and y coordinates in the data (x_1, y_1), ---, (x_n, y_n). This measure is called the correlation coefficient. It is usually denoted by r and is calculated using the following expression:

$$r = \frac{\dfrac{\sum_{i=1}^{n} x_i\, y_i}{n} - \bar{x}\,\bar{y}}{\sigma_x\, \sigma_y} \qquad (1)$$

Where $\bar{x}$, $\bar{y}$ are the means of x_i's and y_i's respectively, σ_x, σ_y, are the standard deviations of the x_i's and y_i's respectively; n is the number of items.

The correlation coefficient is related to the regression line. Recall that the equation of the regression line is given by

$$y = mx + b$$

It can be shown that in terms of σ_x, σ_y, $\bar{x}$, $\bar{y}$, and r, the equation of the regression line can be written as

$$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$$

From this expression it follows that

$$m = \frac{r\sigma_y}{\sigma_x}$$

Therefore

$$r = \frac{m\sigma_x}{\sigma_y}$$

As an example, we shall calculate r using (1). Suppose we have the following data

x	20.4	19.7	21.8	20.1	20.7
y	9.2	8.9	11.4	9.4	10.3

The solution is as follows:

1. Find the mean and standard deviation of x's and record them

   $\bar{x} = 20.54 \quad \sigma_x = .711617875$

2. Find the mean and standard deviation of y's and record them

   $\bar{y} = 9.84 \quad \sigma_y = .9090654542$

3. Find $\bar{x}\,\bar{y}$ and $\sigma_x\,\sigma_y$ and record them

   $\bar{x}\,\bar{y} = 202.1136; \sigma_x\,\sigma_y = .6469072268$

4. Now notice that $\frac{\Sigma x_i y_i}{n}$ is the mean of the products.

A calculator key sequence for this procedure is as follows:

Enter	Press	Display	Comments
20.4	[×]	20.4	
9.2	[=] [Σ+]	1.	$x_1 \cdot y_1$
19.7	[×]	19.7	
8.9	[=] [Σ+]	2.	
.	.	.	
.	.	.	
.	.	.	
20.7	[×]	20.7	
10.3	[=] [Σ+]	5.	Memory is set up to take mean
	[2nd] [MEAN] [−]	202.736	
202.1136	[=] [÷]	0.6224	Entry of $\bar{x}\,\bar{y}$
.6469072268	[=]	.9621163193	Entry of $\sigma_x\sigma_y$ and correlation coefficient r is displayed

The value of r measures the "degree of fit" of the given points to the least squares straight line. When $r = \pm 1$ the correlation is said to be exact. When $r = 0$ the variables are said to be uncorrelated.

APPENDIX C
INVERSE FUNCTIONS

A function, f, may be defined as a rule which maps every number, x, of a set of numbers called the domain to a unique number, y, in another set of numbers called the range. This mapping is represented using the following notation

$$f(x) = y$$

Not all functions have the same domain or range. In many cases, the domain or range is restricted to a particular class of numbers, such as only integers or only positive numbers. However, all functions do have one common feature. Each x (member of domain) has only one y (member of range) assigned to it by the function. For a limited number of functions not only does the function assign to each x a unique y but that y is only assigned to that x and no others. For these special functions, there exists an inverse function, denoted f^{-1}, which is the inverse rule of f, i.e., it maps every value y to a unique value x. This mapping is represented as

$$f^{-1}(y) = x$$

Some properties of a function and its inverse are

1. The domain of a function is the range of its inverse function.
2. The range of a function is the domain of its inverse function.

An important result of these features is that the following relationship exists between a function and its inverse

$$x = f^{-1}(y) = f^{-1}(f(x))$$

$$y = f(x) = f(f^{-1}(y))$$

This implies that if one performs a function on a variable x, then performs the inverse function on the result, one returns to the original variable x.

The function and inverse pairs used on the SR-51 are

f	f^{-1}
e^x	ln x
10^x	log x
sinh	$\sinh^{-1}$
tanh	$\tanh^{-1}$

For example, examine the exponential and natural logarithm pair.

Enter	Press	Display	Comment
2	e^x	7.389056099	
	lnx	2.	Taking Inverse function of a function returns you to the original variable.

There are some functions which do not meet the necessary condition that for each y there is assigned a unique x when the complete domain of the function is taken into account. However, if we restrict the domain to only selected values, then the function meets all criteria and the inverse does indeed exist.

Examples of these are periodic functions and even functions. Your SR-51 employs both types.

f	f^{-1}	type	domain
x^2	$\sqrt{x}$	even	$x \geq 0$
y^x	$\sqrt[x]{y}$	even*	$y \geq 0$
sin	$\sin^{-1}$	periodic	$-\frac{\pi}{2} \leq X \leq \frac{\pi}{2}$ or $-90° \leq X \leq 90°$
cos	$\cos^{-1}$	periodic	$0 \leq X \leq \pi$ or $0 \leq X \leq 180°$
tan	$\tan^{-1}$	periodic	$-\frac{\pi}{2} \leq X \leq \frac{\pi}{2}$ or $-90° \leq X \leq 90°$
cosh	$\cosh^{-1}$	even	$X > 0$

These inverse functions on your SR-51 only return results to the domains specified in column four above.

*When x is odd, y^{x} is also odd. However, your SR-51 does not accept negative arguments for this function.

Example 1:

Enter	Press	Display	Comment
−5	x^2	25.	
	$\sqrt{x}$	5.	Returns positive root only

Example 2: Angle:Deg

Enter	Press	Display	Comment
90	sin	1.	
450	sin	1.	360° + 90° or 90° + one period
	INV sin	90.	Result returned to domain specified above.

While 90° and 90° plus one period have the same value of the sine, your SR-51 only returns results in the restricted domain specified above.

APPENDIX D
CONVERSION CONSTANTS USED IN THE SR-51

1 mil	= 25.4 microns
1 inch	= 2.54 centimeters
1 foot	= 0.3048 meters
1 yard	= 0.9144 meters
1 mile	= 1.609344 kilometers
1 mile	= 0.86897624 nautical miles
1 acre	= 43560.0 square feet
1 fluid ounce	= 29.5735295625 cubic centimeters
1 fluid ounce	= 0.0295735295625 liters
1 gallon	= 3.785411784 liters
1 ounce	= 28.349523125 grams
1 pound	= 0.45359237 kilograms
1 short ton	= 0.90718474 metric tons
1 BTU	= 251.9957611111 calorie grams
1 degree	= 1.111111111111 grads
1 degree	= 0.01745329251994 radians
π	= 3.141592653590

Temperature conversion

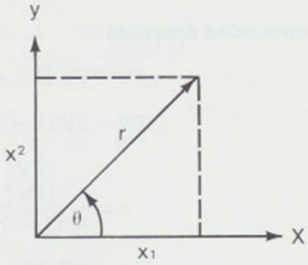
$$°C = \frac{1}{1.8}[°F - 32°]$$

$$°F = 1.8°C + 32$$

Polar-Rectangular

x_1 — first entry

x_2 — 2nd entry



$$r^2 = x_1^2 + x_2^2$$

$$x_1 = r\cos\theta$$

$$\theta = \tan^{-1}\frac{x_2}{x_1}$$

$$x_2 = r\sin\theta$$

Ratio-Decibels

$$\text{Ratio dB} = 20\log\left[\frac{x_1}{x_2}\right]$$

$$\frac{x_1}{x_2} = 10^{\left(\frac{\text{Ratio dB}}{20}\right)}$$

APPENDIX E
FINANCIAL EQUATIONS

Compounded Amounts

$$FV = PV\,(1+i)^n$$

$$PV = FV\,(1+i)^{-n}$$

$$n = \frac{\ln\left[\dfrac{FV}{PV}\right]}{\ln\,(1+i)}$$

$$i = \left[\frac{FV}{PV}\right]^{1/n} - 1$$

where

FV = Future Value

PV = Present Value

n = number of periods

i = interest per period n expressed as a decimal

Annuities

$$FV = PMT\left[\frac{((1+i)^n - 1)}{i}\right]$$

$$PMT = FV\left[\frac{i}{((1+i)^n - 1)}\right]$$

$$n = \frac{\ln\left[1 + FV\left[\dfrac{i}{PMT}\right]\right]}{\ln\,(1+i)}$$

$$PV = PMT\left[\frac{(1-(1+i)^{-n})}{i}\right]$$

$$n = \frac{-\ln\left[1 - PV\left[\frac{i}{PMT}\right]\right]}{\ln(1+i)}$$

$$PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$$

where

FV = Future Value

PV = Present Value

n = number of periods

i = interest per period n expressed as a decimal

PMT = Payment per period n

Remaining Balance

$$Bal_k = PMT\left[\frac{1-(1+i)^{k-n}}{i}\right]$$

Cumulative Interest

$$Int_k = k(PMT) - (PV - Bal_k)$$

where

n = total number of periods

PMT = payment per period

i = interest per period n expressed as a decimal

k = Current period

Bal_k = Balance after k^{th} payment

Int_k = Cumulative interest paid after k^{th} payment

PV = Present Value

APPENDIX F
MATHEMATICAL EXPRESSIONS

Quadratic Equation

If $ax^2 + bx + C = 0 \qquad a \neq 0$

then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Law of Exponents

$$a^x \times a^y = a^{x+y} \qquad \frac{1}{a^x} = a^{-x}$$

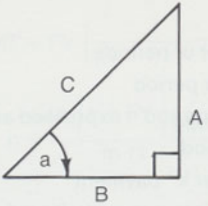
$$(ab)^x = a^x \times b^x \qquad \frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy} \qquad a^0 = 1$$

The following is a very brief review of trigonometric, logarithmic, and hyperbolic functions.

Trigonometric Functions

Trigonometric functions can be defined geometrically in terms of a right triangle.



If the angle a is opposite side A, b is opposite B, and c opposite C, then

$$\sin a = A/C, \ \cos a = B/C, \ \tan a = A/B$$

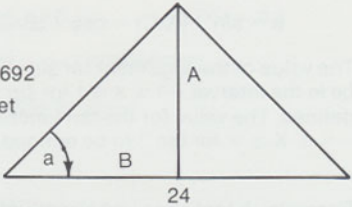
Example: If you are building a roof on a tool shed 24 feet wide, and want to have an angle of 30° to provide drainage then B = 12 feet and a = 30°

$A = B \tan a$

$A = 12 \times \tan 30$

$= 12 \times .5773502692$

$= 6.92820323$ feet



Basic relations for the trigonometric functions are:

$$\sin a = \frac{1}{\csc a};\cos a = \frac{1}{\sec a};\tan a = \frac{1}{\cot a}$$

$$\sin^2 a + \cos^2 a = 1$$

Valid also for any plane triangle

$$A/\sin a = B/\sin b = C/\sin c$$

$$C^2 = A^2 + B^2 - 2\,AB \cos c$$

From calculus the functions can be defined as a series expansion

$$\sin a = a - \frac{a^3}{3!} + \frac{a^5}{5!} - \frac{a^7}{7!} + \ldots$$

$$\cos a = 1 - \frac{a^2}{2!} + \frac{a^4}{4!} - \frac{a^6}{6!} + \ldots$$

$$\tan a = a + \frac{a^3}{3} + \frac{2a^5}{15} + \frac{17a^7}{315}$$

$$+ \frac{62a^9}{2835} \ldots (a^2 < \pi^2/4)$$

where the angle a is expressed in radians.

Inverse Trigonometric Functions

Each function returns the value of the angle if the ratio for the two sides of the triangle is known.

$$a = \sin^{-1}(A/C) = \cos^{-1}(B/C) = \tan^{-1}(A/B)$$

The value of the argument for sin and cos functions must be in the interval $-1 \leq X \leq 1$ for $\sin^{-1}$ and $\cos^{-1}$ to be defined. The value for the tan function is the interval $-\infty \leq X \leq \infty$ for $\tan^{-1}$ to be defined.

Example: A tool shed has a width of 8 feet and a height of 3 feet. Find the angle a of the roof. A = 3, B = 4.

$$\begin{aligned} a &= \tan^{-1}(A/B) \\ &= \tan^{-1}(3/4) \\ &= 36.86989765^\circ \end{aligned}$$

Logarithms

Any positive number can be represented by another positive number, called a base, raised to an appropriate power, an exponent, $x = b^y$. The exponent to which the base must be raised is called the logarithm of the number x, for that specific base, b, Or, $y = \log_b x$. Two bases normally used are 10 and e = 2.718281828. The relationship between these bases can be expressed as:

$$\begin{aligned} \log_{10} x &= \frac{\log_e x}{\log_e 10} \\ &= \frac{\ln x}{\ln 10} \\ &= \frac{\ln x}{2.302585093} \end{aligned}$$

Base 10 logarithms are called common logarithms and base e logarithms are called natural logarithms. Logarithms for negative numbers are undefined.

Example: Determine the time it requires for a radioisotope to decay to 0.1 its present value

$$t = k \ln (X/Xo) \qquad \text{where } k = -1.386/\text{year}$$

$$t = -1.386 (\ln 0.1) \qquad X/Xo = 0.1$$

$$= -1.386 \times (-2.302585093)$$

$$= 3.191382939 \text{ years}$$

Laws of Logarithms

$$\ln(y^x) = x \ln y$$

$$\log(ab) = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

Exponential Functions

Exponential functions occur frequently in the mathematical problems of biology, physics, chemistry and engineering. The value of e^x given by the series expansion is:

$$e^x = 1 + X + \frac{X^2}{2!} + \frac{X^3}{3!} + \frac{X^4}{4!} + \ldots$$

The value of e can be evaluated by allowing $X = 1$: $e = 2.718281828$.

Trigonometric functions can be expressed as functions of e^x.

$$\sin x = \frac{1}{2i}(e^{ix} - e^{-ix}), \qquad i = \sqrt{-1}$$

$$\cos x = \frac{1}{2}(e^{ix} + e^{-ix}).$$

$$\tan x = \frac{(e^{ix} - e^{-ix})}{i(e^{ix} + e^{-ix})}$$

Hyperbolic Functions

Hyperbolic functions may be defined as functions of exponentials

$$y = \sinh x = \frac{e^{x} - e^{-x}}{2},\ y = \cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$y = \tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

The function $y = a \cosh (x/b)$ is known as a catenary and describes the way a power cable, chain, or clothes line supported only by the ends will hang.

Example: The length of a uniform power cable strung between two utility poles of equal height is given by the expression

$$L = (2\ T/W) \sinh (WX/T)$$

where 2X is the distance between poles, W is the weight/ft of cable, and T is the tension at the lowest point. If $X = 45$, $W = 0.78$ lbs/ft and $T = 62$ lbs

$$L = [(2)\ (62)/(.78)] \sinh [(0.78)\ (45)/(62)]$$

$$= 94.88516293 \text{ ft}$$

Basic relations for the hyperbolic functions are:

$$\operatorname{csch} x = 1/\sinh x,\ \operatorname{sech} x = 1/\cosh x$$

$$\tanh x = \sinh x/\cosh x = 1/\coth x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh x = -\sinh (-x),\ \cosh x = \cosh (-x)$$

$$\tanh x = -\tanh (-x)$$

The power series definition of the hyperbolic functions are:

$$\sinh x = x + x^3/3! + x^5/5! + \ldots$$

$$\cosh x = 1 + x^2/2! + x^4/4! + x^6/6! + \ldots$$

$$\tanh x = x - x^3/3 + 2x^5/15 - 17x^7/315 + \ldots \quad (x^2 < \pi^2/4)$$

Inverse Hyperbolic Functions

The relationship between hyperbolic and inverse hyperbolic functions are given by the following expressions:

If $y = \sinh x$, then $x = \sinh^{-1} y$

Example: Determine the distance 2X between the supports for a power cable where W = 0.78 lbs/ft. T = 62 lbs. and the sag in the line is S = 13.082 ft.

$$2X = 2\ T/W\ \cosh^{-1}(WS/T + 1)$$

$$= [(2)\ (62)/(.78)]\ \cosh^{-1}\{[(.78)\ (13.082)/62] + 1\}$$

$$= 90.00077378 \text{ ft.}$$

The basic relations and identities for the inverse hyperbolic function are:

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) = \cosh^{-1}(\sqrt{x^2 + 1})$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) = \sinh^{-1}(\sqrt{x^2 - 1})$$

$$\tanh^{-1} x = 1/2 \ln[(1 + x)/(1 - x)]$$

SERVICE INFORMATION

Battery Pack Replacement

The battery pack can be quickly and simply removed from the calculator. Hold the calculator with the keys facing down. Place a small coin (penny, dime) in the slot in the bottom of the calculator. A slight prying motion with the coin will pop the slotted end of the pack out of the calculator. The pack can then be removed entirely from the calculator.

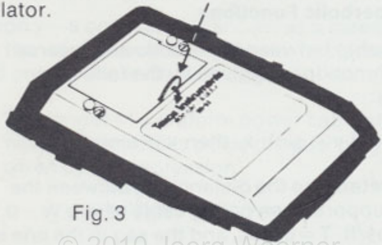


Fig. 3

The exposed metal contacts on the battery pack are the battery terminals. Care should always be taken to prevent any metal object from coming into contact with the terminals and shorting the batteries.

To re-insert the battery pack, place the rounded part of the pack into the pack opening so that the small step on the end of the pack fits under the edge of the calculator bottom. The slotted end of the pack will then be next to the instruction label. A small amount of pressure on the battery pack will snap it properly into position.

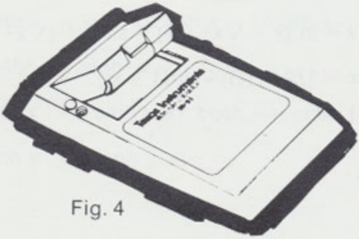


Fig. 4

Spare and replacement battery packs can be purchased directly from a Texas Instruments Consumer Services Facility as listed on the back cover.

AC Adapter/Charger

Battery pack recharge or direct operation from standard voltage outlets is easily accomplished with the AC Adapter/Charger model AC9200 or AC9130 included with the SR-51 (also used with the SR-10, SR-11, SR-16, and SR-50). The SR-51 cannot be overcharged; it can be operated indefinitely with the adapter/charger connected.

Operating Conditions

CAUTION: Before recharging, check to make sure the battery pack is properly installed and that the switch on the adapter/charger (AC9200 only) is set at the line voltage corresponding to your AC outlet.

Recharge the battery pack when the display flashes erratically or fades out.

To prolong operating time before the next recharging, press $\boxed{\text{C}}$ after the desired answers have been displayed. Turn your SR-51 off when not in use.

Battery Operation

The "fast-charge" nickel-cadmium battery pack BP-1 furnished with the SR-51 calculator was fully charged at the factory before shipping. However, due to shelf life discharging, it may require charging before initial operation.

With the battery pack properly installed in the bottom of the SR-51, charging is accomplished by plugging the AC Adapter/Charger AC9200 or AC9130 into a convenient outlet and plugging the attached cord into the SR-51 socket. A full charge will take approximately 4 hours with the calculator off.

If the SR-51 is left on for an extended period of time after the batteries become discharged, one of the batteries may be driven into deep discharge. This condition is indicated by failure of the calculator to operate after being recharged for a few minutes. The batteries can usually be restored to operating condition by charging the calculator overnight. Repeated deep discharging will permanently damage batteries.

In Case of Difficulty

1. Check to be sure calculator is correctly plugged into a proper outlet that has power and that the AC Adapter/Charger voltage switch (AC9200 only) is set on the correct voltage.
2. Check to be sure ON-OFF switch is in the ON position. Presence of digits in the display indicates power is on.
3. Press [2nd] [CA] and re-enter problem.
4. If display fails to light on battery operation, recharge batteries.
5. Review operating instructions to be certain calculations are performed correctly.

If none of the above steps correct the difficulty, return the unit *with charger*, battery pack and packing material, postpaid for repair to your nearest Texas Instruments Consumer Service Facility listed on the following page. Please include information on your difficulty as well as return information of name, address, city, state and zip code.

CAUTION: Use of other than the AC Adapter/Charger AC9200 or AC9130 may apply improper voltage to your SR-51 calculator and will cause damage.

If You Have Questions or Need Assistance

If you have questions or need assistance with your calculator, write the Consumer Relations Department at:

Texas Instruments Incorporated
P. O. Box 22283
Dallas, Texas 75222

or call Consumer Relations at 800-527-4980 (toll-free within all continental states except Texas) or 800-492-4298 (toll-free within Texas). If outside continental United States call 214-238-5461. (We regret that we cannot accept collect calls at this number.)

Warranty Registration

To protect your warranty, complete and mail the attached Warranty Registration Card within 10 days of purchase or receipt as a gift. Also record the serial number of your calculator below. Any correspondence concerning your calculator must include both model and serial number.

SR-51		
Model No.	**Serial No.**	**Purchase Date**

1 ☐ Mr.
2 ☐ Miss
3 ☐ Mrs.

Warranty Registration Card

Mail within 10 days to protect your warranty

			SR-51		
Owner's First Name	**Initial**	**Last Name**	**Model No.**	**Serial No.**	**Purchase Date**

Owner's Mailing Address	**City**	**State**	**Zip**

Please help us in planning other useful products by providing the following information:

Was Your TI Calculator a Gift?
1 ☐ Yes 2 ☐ No

Where the Calculator Will Be Used (Check One)
1 ☐ Home
2 ☐ Occupation
3 ☐ Both

Where Purchased?
1 ☐ Department Store
2 ☐ Office Equipment Dealer
3 ☐ Mail Order
4 ☐ Other (Specify)

Your Occupation (Check One)
1 ☐ Engineer/Scientist
2 ☐ Salesman
3 ☐ Accountant
4 ☐ Farmer/Rancher
5 ☐ Student
6 ☐ Educator
7 ☐ Homemaker
8 ☐ Doctor/Lawyer
9 ☐ Banker/Financier
10 ☐ Other (Specify)

Your Approximate Age
1 ☐ Under 18
2 ☐ 18-24
3 ☐ 25-34
4 ☐ 35-54
5 ☐ 55 and over

Your Approximate Yearly Family Income
1 ☐ Under $5000
2 ☐ $5,000 to $10,000
3 ☐ $10,000 to $15,000
4 ☐ Over $15,000


SR-51

PLACE
STAMP
HERE

TEXAS INSTRUMENTS INCORPORATED
P.O. BOX 5012
DALLAS, TEXAS 75222

Texas Instruments
super slide-rule calculator
SR-51

ONE YEAR WARRANTY

The SR-51 electronic calculator from Texas Instruments is warranted to the original purchaser for a period of one year from the original purchase date — under normal use and service against defective materials or workmanship.

Defective parts will be repaired, adjusted and/or replaced at no charge when the calculator is returned prepaid to a Texas Instruments Consumer Service Facility listed below.

The warranty is void if the calculator has been visibly damaged by accident or misuse, if the serial number has been altered or defaced, or if the calculator has been serviced or modified by any person other than a Texas Instruments Consumer Service Facility.

This warranty contains the entire obligation of Texas Instruments Incorporated and no other warranties expressed, implied, or statutory are given.

The warranty is void unless the attached Warranty Registration Card has been properly completed and mailed to Texas Instruments Incorporated within 10 days of purchase.

Texas Instruments Consumer Services Facilities

Mailing Address:
Texas Instruments Service Facility
P.O. Box 22283
Dallas, Texas 75222

Canadian Address:
Texas Instruments Service Facility
41 Shelley Road
Richmond Hill, Ontario, Canada

Consumers in California and Oregon may contact the following Texas Instruments offices for additional assistance or information:

Texas Instruments Consumer Service
78 Town and Country
Orange, California 92668
(714) 547-2556

Texas Instruments Consumer Service
10700 Southwest Beaverton Highway
Park Plaza West Suite 111
Beaverton, Oregon 97005
(503) 643-6758

TEXAS INSTRUMENTS
INCORPORATED
DALLAS, TEXAS

PRINTED IN U.S.A. 1220619-1